Building Credit Histories[*]

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December 2024

Abstract

This paper investigates how new borrowers expand their credit access. In particular, we examine the role that consumers' credit choices, not just repayment behavior, play in building their credit histories. Using credit-bureau data, we document that incumbent lenders typically increase credit limits for borrowers who open additional credit cards. This effect is especially pronounced for new borrowers. Our interpretation of this evidence is that lenders perceive credit offered by other lenders as revealing favorable information about the borrower. We build a novel model consistent with this hypothesis and show that the model's predictions are consistent with the data.

Keywords: Emerging Borrowers, Credit History, Information Aggregation, Debt Dilution.

JEL Codes: D14, D82, D83, D86, G21

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[*]We have benefited from comments and suggestions by Mitchell Berlin, Dean Corbae, Andrew Glover, Andrei Kovrijnykh, Alvin Murphy, Chris Phelan, Greg Schlichter, seminar participants at ASU, Bank of Canada, Drexel University, Federal Reserve Bank of Philadelphia, IntraFed, Hunter College CUNY, McMaster University, Nazarbayev University, Stony Brook University, UC Davis, UCSB, University of Colorado Boulder, University of Wisconsin, Western, Wharton Macro Lunch, and conference participants at BEROC Academic Conference 2022, Band of Canada–Bank of Japan–Philadelphia Fed Joint Workshop 2024, Conference in Memory and Honor of Robert E. Lucas, Jr., NBER "Innovative Data in Household Finance: Opportunities and Challenges" 2022, NBES 2023, SAEe 2024, SAET 2022, SED 2023, System Banking 2024, and Women in Macro 2024. The authors gratefully acknowledge financial support from the National Science Foundation, grant #2019160. Zetlin-Jones is grateful to the PNC Center for Financial Services Innovation for financial support. Parts of the work on this paper were carried out when Livshits and Zetlin-Jones were visiting CASEE; they thank the center for its kind hospitality. We thank Jacob French, Nicholas Hoffman, and Jacob Shepard for excellent research assistance. The views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

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1 Introduction

Credit access is important for socio-economic well-being, as it facilitates households' ability to withstand economic shocks and smooth consumption over time. An individual's access to credit is in large part determined by their credit history. But how does one start building a credit history? This paper investigates, both theoretically and empirically, how new borrowers acquire and expand their access to credit. Whereas the literature typically thinks of lending decisions relying largely on borrowers' repayment histories, when it comes to new borrowers, such histories are short. Thus, lenders may rely on other information when lending to new borrowers. We explore how lenders react when they observe that a borrower obtained a new loan from another lender. In particular, we document that incumbent lenders typically increase credit limits to borrowers who open additional credit cards. Crucially, this increase is especially pronounced for new, or emerging borrowers—those borrowers who have only recently obtained their first credit product.

One interpretation of this evidence is that lenders perceive credit offered by other lenders as revealing new, favorable information about a borrower. To capture this idea, we build a simple model in which different lenders have different information about a borrower's creditworthiness. We use this model to analyze the role that the borrower's credit history plays in aggregating this dispersed information. We derive testable implications of this theoretical mechanism and show that they are consistent with the data.

We start by documenting key facts about emerging borrowers using a novel data set from a credit reporting agency, TransUnion. We define an *emerging borrower* as a borrower whose oldest line of credit is at most 6 months old. Our data set has two important features. First, it over-samples emerging borrowers. Second, it enables us to examine the evolution of individual credit lines over time. We use these features to contrast the evolution of credit access of emerging borrowers against that of established borrowers, and shed light on the importance of borrowing from multiple creditors.

We document the following observations. First, emerging borrowers who open a new credit card see a sizable increase in the credit limit on their original credit card. In fact, the increase in the credit limit on existing credit cards is larger for borrowers who open a new card than for those who do not. Moreover, the incumbent lender's contribution to the total credit-limit growth of an emerging borrower opening a new card is larger than that of the new lender. This is not the case for established borrowers.

That is, incumbent lenders react positively to seeing a new card, and do so more for emerging than for established borrowers. Second, the timing of this increase in the credit limit coincides almost exactly with the timing of the new card appearing on the borrowers' credit record, before any repayment history on the new card can be established. While other forces—such as borrowers' increased demand for credit or a lender's desire to be the borrower's primary credit-card provider—might be at play, we argue that they do not fully explain our empirical observations.

Our data findings lead us to hypothesize that borrowing from one lender may lead to an improved assessment of the borrower's creditworthiness by other lenders. We refer to this mechanism as "building a credit history." The idea is as follows. When lenders rely on private information to advance loans, a publicly recorded accepted loan reveals some of that private information. Assuming that lenders with more positive information are more likely to offer loans, other lenders who see a a loan will increase their own assessment of the borrower and offer better subsequent terms of credit. We develop a new theory consistent with this hypothesis which allows us to better understand and evaluate tradeoffs associated with credit-history building and to derive testable implications.

Our theory features multiple competing lenders who have heterogeneous private information about a consumer's creditworthiness. The assumption of information heterogeneity across lenders warrants some discussion. This heterogeneity may come from the lenders' use of different statistical models and/or differences in their information (beyond what is contained in the public credit records). Indeed, lenders spend considerable resources on developing and improving their models of credit-risk assessment, referred to as scorecards.[1] These scorecards are distinct from general purpose credit scores such as FICO and often rely on information beyond that available from credit bureaus (see discussion and references in Livshits et al., 2016, Section 2.1). Furthermore, the use of alternative data in credit-risk assessments has grown dramatically in recent years. The examples of such alternative data include debit/credit transactions, rental and utility data, clickstream data, social profile and social networks data, and text data.[2] Such alternative data sources are particularly important for emerging borrowers who have little or no formal credit history.[3] To sum up, the use of different data sources and statistical models results in distinct assessments of borrowers' creditworthiness across lenders.[4]

[1]Chandler (2004) estimates the cost of developing and implementing a single scorecard between $40,000 and well over $100,000. Large lenders may have as many as 80 different scorecards.

[2]For further discussion see, e.g., https://www.fico.com/blogs/how-use-alternative-data-credit-risk-analytics and https://www.mckinsey.com/capabilities/risk-and-resilience/our-insights/designing-next-generation-credit-decisioning-models.

[3]For example, Chioda et al. (2024) use data from a large fintech lender in Mexico and show that alternative data from digital transactions through a delivery app are effective at predicting creditworthiness for borrowers with no credit history.

[4]Naturally, lenders are aware of this disperse information. In support of this point, Balyuk (2023) documents that traditional lenders view fintech loans (peer-to-peer lending) as informative of the borrowers' creditworthiness.

We investigate how the dispersed private information is aggregated through lending that takes place over multiple stages. The model has two periods: the first, lending period and the second, repayment period. The lending period consists of two stages. Each borrower has zero income in the first period and uncertain income in the second period. In the beginning of the first period, lenders receive private signals about the distribution of the borrower's income in the second period. For simplicity, we assume that signals are binary and are either positive or negative. Lenders offer loan contracts—described by the loan size and price—to the borrower in each of the two stages of the first period.[5] Lenders do not observe each others' offers, but they observe the contract that the borrower accepts.

We analyze Perfect Bayesian Equilibria of this game. As most signaling models, ours features multiple equilibria and so we employ an equilibrium selection in the spirit of the Cho and Kreps (1987) intuitive criterion and the Miyazaki-Wilson allocation, which selects the equilibrium (outcome) most preferred by the least risky type of borrowers.[6]

Credit-history building arises in equilibrium as follows. Borrowers who see offers from lenders in the first stage conclude that these lenders have positive signals about them, because negatively informed lenders do not make offers in the first stage. To transmit this information to other lenders, these borrowers accept an offer—i.e., take out a loan—from a positively informed lender in the first stage. Lenders who see that a borrower accepted an offer conclude that this offer came from a lender with a positive signal, update their belief about the borrower's creditworthiness upwards, and offer better contract terms in the second stage.

Our model yields a novel testable implication related to debt dilution. As in other models with borrowing from multiple lenders, our equilibria feature debt dilution—taking an additional loan decreases the probability of repayment of the initial loan. However, our model generates a counterintuitive prediction that we refer to as "more dilution, lower default risk:" when the original lender faces uncertainty about how much his early loan will be diluted, he is actually *more* likely to be repaid when the borrower accepts a *larger* additional loan from another lender. In other words, the more the lender's initial loan is diluted ex post, the more likely the lender is to be repaid, all else equal. There are two competing forces at play. First, as we already pointed out, our mechanism has a "dilution effect:" for a borrower of a given risk/quality, a larger loan increases the probability

[5]We discuss how our model can be extended to include a distinction between credit balances and credit lines in Appendix B.

[6]The basic idea of the so-called Miyazaki-Wilson allocation goes back to Miyazaki (1977) and Wilson (1977). Netzer and Scheuer (2014) provide both a very elegant explicit game-theoretic microfoundation for this equilibrium allocation and a nice review of the history of thought on this subject (including Riley, 1979, Engers and Fernandez, 1987, and Hellwig, 1987, among others).

of default. There is, however, also a "selection effect:" in equilibrium, less risky/better quality borrowers take out larger loans. This selection effect dominates the dilution effect. That is, a better quality borrower taking a larger additional loan is less likely to default than a worse quality borrower taking a smaller additional loan. Importantly, information aggregation is key for this result: a larger additional loan conveys positive information of the diluting lender about the borrower's creditworthiness.

This counterintuitive prediction is actually borne out in the data. We show that, unconditionally, opening a new credit card increases the probability of future delinquency as in standard models of debt dilution. However, among borrowers who open an additional credit card, their probability of future delinquency is *decreasing* in the size of their new card's credit limit, as our model predicts. That is, in the data, our novel observation that more dilution is associated with a lower default risk coexists with the conventional one that dilution increases default risk, just as in our model.

Debt dilution makes credit-history building potentially costly: the least risky borrowers may end up with excessively large loan obligations (relative to the symmetric-information benchmark). Debt dilution and excessive borrowing are common features of models with sequential borrowing (see, e.g., Bizer and DeMarzo, 1992), but they can be entirely avoided in our model by borrowing only in the second stage. The only reason for overborrowing in our model is credit-history building.

We explore whether the least risky borrowers prefer equilibria with credit-history building to equilibria without credit-history building, where no offers are made in the first stage. We show that when the cost of excessive borrowing is particularly severe (which happens on a small set of parameter values when computed numerically), the selected equilibrium features no credit-history building.

Notably, our equilibrium selection picks credit-history building on a larger set of parameter values than what is desirable from the borrowers' ex-ante perspective (before the signals are realized).[7] The reason is as follows. When the least risky borrowers choose to build a credit history, they improve their own borrowing terms but worsen them for the types of borrowers they are no longer pooled with. Sometimes, borrowers would ex ante prefer (to commit) not to build credit history but ex post the least risky borrowers find it in their interest to do so.

Thus, our model suggests an important welfare implication: publicly recording borrowers' credit histories is not always desirable from social welfare perspective.[8] Availability of credit records allows lenders to tailor loans based on more precise information,

[7]Lenders in our model break even, so the social welfare is equal to the borrower's expected utility.
[8]Note that our welfare analysis ignores the role of credit records as histories of debt *repayment*.

but it may lead to excessive borrowing by the highest quality borrowers. When the cost outweighs the benefits, availability of credit records reduces ex-ante social welfare.

Our model also allows us to think about the following related question: In the presence of credit records, does more precise information—e.g., arising from an improvement in lenders' statistical models—make borrowers better off? We show that the ex-ante welfare can be non-monotone in the quality of information. While generally welfare rises with the signal quality, it can drop discontinuously in some cases. The reason the welfare can drop is, once again, excessive borrowing. As signals become more precise, the least risky borrowers get better terms on any given-size loan in the second stage. Sometimes these better terms lead them to overborrow, which results in a drop in their ex-ante utility.

It is worth noting that in our model, there is an important distinction between building a credit history and improving a credit score. Credit scores are meant to be a summary statistic for borrowers' probability of default. Building a credit history in our model may actually lower a borrower's credit score.[9] Borrowers who take on early loans successfully communicate that they have a lower default probability *for a given loan size*, but they also end up with a higher default probability in equilibrium due to taking on a *larger* loan.[10]

This rest of the paper is organized as follows. The next subsection discusses related literature. Section 2 presents some empirical facts regarding emerging borrowers, as well as suggestive evidence for the information-aggregation mechanism we focus on. Section 3 presents a simplified version of the model to illustrate the mechanism of credit-history building. Section 4 analyzes a more general model that allows us to deliver additional insights. Section 5 discusses the novel prediction about debt dilution and shows that it is borne out in the data. Section 6 concludes. Additional empirical analyses, model extensions, a numerical example, omitted proofs, and details of equilibria constructions are in the Appendices.

1.1 Related Literature

To date, research on consumer credit has predominantly focused on the middle-to-end of a consumer's credit life cycle, with empirical work by Brevoort and Kambara (2017) and Santucci (2019) being rare and welcome exceptions. When it comes to credit records and credit histories, the existing literature has focused on the impact of borrowers' *repayment* behavior on subsequent access to credit (see Chatterjee et al., 2016 and Kovbasyuk et al.,

[9]The insight that opening a new card may lower one's credit score is consistent with the discussion of the impact of new credit on credit scores on FICO's website Fair Isaac Corporation (2022).

[10]Correspondingly, the mechanism we are highlighting is distinct and complementary to the idea of doctoring one's credit score, as in, for example, Hu et al. (2017).

2018 for leading examples, and Livshits, 2015 and references therein for a wider literature review). In contrast, we focus on emerging borrowers and the importance of the record of their *borrowing* for the evolution of their access to credit.

Our focus on information aggregation yields novel insights into debt dilution. A key feature of our model is non-exclusivity of relations between borrowers and lenders. Although a large literature has examined consumer credit markets, it has typically assumed exclusivity of debt contracts—see, e.g., Chatterjee et al. (2007), Livshits et al. (2007), and surveys by Athreya (2005) and Livshits (2015). While debt dilution is a prominent feature of recent papers on defaultable debt in international finance—see, e.g., Aguiar et al. (2019), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012, 2015)—the questions studied in that literature are very different from those in the consumer credit literature. The idea of information aggregation among lenders is new to either literature and constitutes our central contribution.

Our paper also provides a theory of why borrowers take loans from multiple lenders. This important feature is absent, for example, from a seminal paper by Bizer and DeMarzo (1992), which shows that the anticipation of debt dilution leads to a too large loan at a too large interest rate, but the whole loan may as well be originated by a single lender. Parlour and Rajan (2001) provide a theory of borrowing from multiple lenders, but in their model borrowing is not sequential, and there is no credit-history building, which is the focus of our paper.[11]

The concept of information aggregation is related to the quality of information available to lenders. Narajabad (2012), Sanchez (2018), Athreya et al. (2012), Livshits et al. (2016), Drozd and Serrano-Padial (2017) investigate the implications of improvements in the quality of (public) information in consumer credit markets for aggregate outcomes in the unsecured credit market.[12] These papers treat the information improvements as exogenous (arising from IT revolution, better quality data, or improved credit-scoring models), while we treat this information as endogenous and model it as an outcome of (strategic) behavior of borrowers.

The idea of learning from actions of others (as lenders do in our environment) is, of course, not unique to our setting—see Bikhchandani et al. (1999) for a nice discussion of informational cascades in various applications. Ruckes (2004) and Dell'Ariccia and Mar-

[11]Borrowing from multiple lenders is also absent from the quantitative-theory literature on competition in consumer credit markets (e.g., Drozd and Nosal, 2008, Galenianos and Nosal, 2016, Galenianos et al., 2021) with an exception of Herkenhoff and Raveendranathan (2020), in whose model every borrower gets credit cards from all the lenders in the economy.

[12]In addition to the quantitative-theory papers listed here, it is worth pointing to important empirical papers by Musto (2004) and Liberman et al. (2018), which investigate the effects of information *leaving* public credit records.

quez (2006) study endogenous quality of information that lenders obtain about prospective borrowers and the evolution of lending standards. These models feature competition among heterogeneously-informed lenders, but the borrowers are restricted to accepting loans from just one lender.

We find that greater precision of information does not always improve ex-ante welfare, and may even lower utility of the borrowers with the highest signals. The idea of non-monotonicity of welfare in the precision of information goes back to Hirshlelfer (1971) and appears in a wide range of environments and applications: see Padilla and Pagano (2000) for credit records, Andolfatto (2010) and Andolfatto et al. (2014) for monetary economies with matching frictions, Kaplan (2006), Gorton and Ordoñez (2014) and Dang et al. (2017) for banking, as well as Monnet and Quintin (2017), Pagano and Volpin (2012), Goldstein and Leitner (2018), and Lester et al. (2019), just to name a few.

Ours is not the first paper to explicitly analyze welfare implications of the availability of public credit records. Corbae and Glover (2018) analyze how employers' ability to access credit records affects matching efficiency in the labor market. Elul and Gottardi (2015) and Padilla and Pagano (2000) consider welfare implications of bankruptcy filings being part of a (permanent) public record through their disciplining effect on borrowers. Blattner et al. (2022) analyze optimal information design of credit histories in the presence of adverse selection. They find that efficiency effects of credit reporting depend on the extent of adverse selection (captured by the correlation between borrowers' demand and risk) and the informativeness of credit histories (captured by the persistence of consumers' types). The basic underlying mechanisms in all these papers are very different from ours, as we focus on the record of borrowing, rather than the record of (non-)repayment.

One other paper that explicitly endogenizes information sharing in the consumer credit market is Pagano and Jappelli (1993). Unlike Pagano and Jappelli (1993), who consider lenders' incentives to share information through credit bureaus, we assume that loans are always reported to the credit bureaus and focus on the borrowers' incentives to build credit histories (and on the implications of the availability of credit bureaus for borrowers' welfare).

A recent empirical literature has begun to investigate the role of information sharing across lenders in determining terms of credit. Balyuk (2023) documents an observation quite similar to our key motivating fact—that banks expand access to credit to borrowers who obtain peer-to-peer loans, apparently viewing these fintech loans as positive signals regarding the borrowers' creditworthiness. Studying firms' access to credit, Sutherland (2018) finds empirically that when lenders share information, their relationship with borrowers tends to dissolve more quickly. Hertzberg et al. (2011) also study how information sharing across lenders determines borrowers' terms of credit, highlighting the coordination role of the shared information. These papers treat the nature of information that is shared across lenders as exogenous, while we emphasize the borrowers' incentives to *affect* the information that is shared.[13]

Finally, our paper offers a new way of interpreting some findings of a growing empirical literature, including Liberman et al. (2021), who look at the effects of taking a payday loan on financial outcomes of borrowers in the UK, where such loans are reported to the credit bureaus. The mechanism we are highlighting may help explain why taking on an additional (payday) loan does not lead to any additional financial distress for the borrowers with the lowest ex-ante credit scores.

2 Motivating Empirical Facts

We explore how new borrowers acquire and expand their access to credit using a novel panel data set that we acquired from a credit reporting agency, TransUnion. Our data contain information on the credit activities of one million anonymized individuals over a 4-year period, 2014-2017. (No personally identifiable information was provided to us at any time by TransUnion for the purpose of this research.) In particular, for each individual we observe a snapshot of their credit information as of September 30th of each year. Our data include information on an individual's credit activity (lenders' inquiries, credit limits, balances, loan performance, etc.) across a wide range of credit products, but not assets, income, or demographic characteristics.

Importantly, half of our sample constitutes new entrants to consumer credit markets. Specifically, for these new entrants the oldest trade or line of credit is at most 6 months old in 2014. We call these individuals *emerging* borrowers. The remaining half of our sample, which we refer to as *established* borrowers and use for comparison purposes, excludes emerging borrowers and otherwise constitutes a representative sample of the Vantage Score distribution in 2014.

In this section, we first present basic descriptive statistics of emerging borrowers and compare them to those of established borrowers. We document how—i.e., with what credit products—emerging borrowers enter the credit market, and demonstrate that they have significantly less access to credit relative to established borrowers. We then docu-

[13] Martin (2009) investigates a very different mechanism to address adverse selection. The key similarity to our paper is that the "good" borrowers in that model take loans over two stages from two (possibly distinct) lenders. Furthermore, the early stage loan pools these "good" borrowers with "bad" borrowers, as it does in some equilibria with credit-history building in our model. However, the explicit signaling motive is absent in Martin (2009).

ment the following three key observations: (i) credit growth of emerging borrowers is substantial, particularly in comparison to established borrowers; (ii) both existing and new credit lines play important roles in the expansion of credit for emerging borrowers; (iii) incumbent lenders tend to increase credit limits in response to emerging borrowers' obtaining a new credit card, and this effect is substantially less pronounced for established borrowers. Taken together, these observations suggest that borrowing from multiple lenders is important for emerging borrowers' credit-access expansion, and that credit histories are essential for information aggregation across lenders.

2.1 Descriptive Statistics

Tables 1 and 2 provide descriptive statistics for the borrowers in our sample. Table 1 highlights the set of credit products that facilitate emerging borrowers' entry into the credit market. It illustrates that more than half of the entries into the credit market are via a credit card.[14]

Table 1: Percent of Baseline Sample With Open Credit Types

Percent of sample with...	Emerging	Emerging with credit card	Established	Established with credit card
Auto	13.5	2.9	24.5	33.5
Credit card	52.6	100.0	62.6	100.0
Mortgage	0.4	0.1	25.1	36.4
Retail	14.8	4.1	47.7	66.3
Student loan	13.3	1.9	11.9	14.4
Mean number of open trades	1.179	1.173	5.071	6.668
Mean number of total trades	1.196	1.185	11.313	15.654
Mean age oldest trade (months)	2.7	2.7	195.7	239.8
Number of observations	500,000	263,103	500,000	312,886

Notes: The table displays the percent of each sample with the indicated types of open credit trades, measuring at the baseline observation (2014). Credit types are not mutually exclusive.

Looking at Table 2 further confirms the importance of credit cards, as they account for more than half of total unsecured credit line for both emerging and established borrowers. At the same time, Table 2 highlights important differences between emerging and established borrowers: the average established borrower with a credit card had the overall credit-card limit nearly ten times as large as that of the average emerging borrower with a credit card.[15] Overall, emerging borrowers had an average of $4,600 of non-mortgage

[14] A typical emerging borrower has just one credit account and for over 50% of emerging borrowers this account is a credit card. The first credit product for those without a credit card is roughly equally likely to be an auto loan, a retail trade, or a student loan. Exceedingly few emerging borrowers enter the market with a mortgage or a home equity loan. Emerging borrowers tend to be young, which is reflected in them being more likely to have a student loan and highly unlikely to have a mortgage. For more robust evidence on the demographics of new borrowers see Livshits (2022).

[15] The difference is even larger for the median value—more than a factor of 20, see Table A3.

credit in our first year of observation, 2014, while established borrowers had access to ten times as much credit. Comparing balances with credit lines, we can see that credit utilization is much higher for emerging than for established borrowers.

Table 2: Baseline Credit Lines and Balances

	Emerging	Emerging with credit card	Established	Established with credit card
Credit Line				
All (no mortgage)	4,671	3,531	45,339	49,891
	[495k]	[263k]	[358k]	[307k]
Auto	14,353	15,162	25,937	26,834
	[67k]	[8k]	[123k]	[105k]
Credit card	2,922	2,922	27,301	27,301
	[263k]	[263k]	[301k]	[301k]
Mortgage	229,923	-	218,163	225,835
	[2k]		[126k]	[114k]
Retail	1,392	896	7,103	7,571
	[74k]	[11k]	[217k]	[194k]
Student loan	4,070	4,358	32,691	35,589
	[66k]	[5k]	[59k]	[45k]
Balance				
All (no mortgage)	3,964	1,696	19,541	19,944
	[399k]	[191k]	[329k]	[286k]
Auto	13,953	14,685	17,396	17,824
	[67k]	[8k]	[123k]	[105k]
Credit card	946	946	5,641	5,641
	[186k]	[186k]	[268k]	[268k]
Mortgage	226,089	-	190,502	197,775
	[2k]		[126k]	[114k]
Retail	712	489	1,556	1,592
	[48k]	[7k]	[125k]	[112k]
Student loan	3,980	4,260	30,371	32,376
	[66k]	[5k]	[59k]	[45k]

Notes: The table reports mean amount of credit or balance in USD, measuring at the baseline observation (2014). Numbers of observations are in brackets. Cells representing less than 0.1% of the sample (less than 500 observations) are excluded. Means are conditional on having the credit type. Credit limits and balances are taken from trades verified in the preceding 12 months.

In what follows, we focus on credit cards.[16] There are several reasons for this. First, as we just argued, a credit card is the main entry product for new borrowers. Second, it allows us to observe the (incumbent) lenders' *response* to new information: a change in the credit limit. Third, our model is about non-exclusive unsecured credit, and a credit card is a product that fits this description possibly the best. Lastly, most of the related literature studies credit cards, so it is natural for us to do the same.[17]

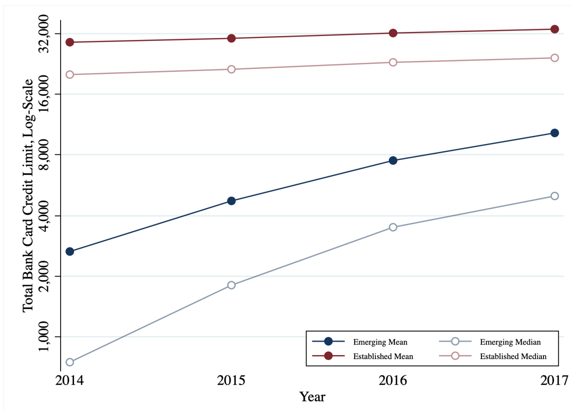
[16]Our definition of a credit card is a "bankcard" in TransUnion's language: it is an open-loop credit card extended by a bank, credit union, or finance company. It excludes retail cards—closed-loop credit cards issued by other kinds of businesses to facilitate purchases at a specific retailer/retailer group.

[17]To further justify the choice of focusing on credit cards, compare statistics for individuals with credit cards in Tables 1 and 2. Table 1 shows that emerging borrowers who had a credit card were much less likely to have other types of credit. However, Table 2 shows that emerging borrowers with both a credit card and a second type of credit had remarkably similar credit limits and balances in those second credit types as the average emerging borrower. This suggests that any selection on unobservables when considering only emerging borrowers with a credit card is unlikely to be quantitatively important.

2.2 Credit Limit Growth

In this subsection, we establish that emerging borrowers' credit grows much faster than that of established borrowers. An average (median) emerging borrower in 2014, their first year of borrowing, has access to credit lines of $2,946 (median $800), whereas the corresponding number for established borrowers is $27,215 (median $18,100). In aggregate, from 2014 to 2015, the average credit limits of emerging borrowers grew 69.9% while that of the established borrowers grew 4.3%. Figure 1 illustrates credit growth of the average and median emerging and established borrowers from 2014 to 2017 and illustrates that growth in credit access is concentrated among emerging borrowers. Despite the much faster growth, even after four years, emerging borrowers have substantially less credit than a typical borrower. That is, being an emerging borrower is not a transient state and seems to last a while. Thus, it is important to study emerging borrowers and how they establish and extend their access to credit.

Figure 1: Credit Growth



Notes: The figure plots the mean and median of the total credit card limit over the four observation periods on a log-scale. The sample is conditional on observing a credit limit in each period. Therefore, the sample is constant within groups across periods.

The results presented above are based on aggregates of total credit across all individuals. At the individual level, our data allow us to break down credit growth arising from incumbent or existing accounts and credit growth arising from the addition of new accounts as presented in the next subsection. In order to do this, we must be able to track individual cards for a specific borrower over time. In Appendix A, we describe the card matching process we use to construct these linkages.

2.3 Credit Limit Growth on New vs. Existing Cards

Table 3 presents growth rates of the aggregate credit limits for emerging and established borrowers, and the decomposition of this growth coming from changes in existing vs. new cards' credit limits. Similar to what we already saw in the previous subsection, the growth rate of the aggregate credit limit of emerging borrowers is much larger than that of established borrowers: 58.69% compared to 2.83%.[18] This fact is not surprising, and it is partly explained by the fact that emerging borrowers start with lower credit limits. However, not only relative, but also absolute increases in the credit limit are larger for emerging borrowers compared to established ones.[19]

Table 3: Growth Rate of Aggregate Credit Limit from 2014 to 2015, %

	Emerging	Established
Total	58.69	2.83
Cond. new card	226.38	31.32
Cond. no new card	22.58	−2.55
Incumbent cards, cond. new card	142.60	5.75

Notes: The sample includes all borrowers with non-missing credit limits in 2014 and 2015 and with no more than five cards in 2015, or borrowers with non-missing credit limits in 2014 who have zero cards in 2015.

Next, note that the overall credit limit growth is much higher among borrowers who open a new card than among those who do not. Specifically, the credit limit increase for emerging borrowers who open a new card is 226.38% compared to 22.58% for those who do not open a new card.[20] The numbers for established borrowers are 31.32% and −2.55%, respectively. The higher growth for borrowers with new cards is not surprising,

[18]The sample for which we report the numbers below includes only individuals who had no more than five cards in 2015. This restriction allows us to correctly attribute credit limit to new and old cards, as we have detailed information for only five cards per borrower. We further require that the total credit limit is observed in 2014 and in 2015 (unless a borrower has no cards in 2015). Table A4 in Appendix A also presents numbers for the less restrictive sample (Sample 1), which includes borrowers with more than five cards in 2015 and thus matches the aggregate statistics reported above (but forces us to make assumptions about the age of cards we do not observe), and the more restrictive sample (Sample 3), which requires that we observe the credit limit of every single card a borrower has in 2015. The sample presented here corresponds to Sample 2 in Table A4. The key patterns reported here hold for each of these samples, even as the levels of growth vary across the samples. The difference in the total credit limit growth rates relative to subsection 2.1 is due to a different sample selection.

[19]Table A5 in Appendix A shows that the increase in the average credit limit for emerging borrowers in dollar terms is 2.5 times larger than the corresponding increase for established borrowers.

[20]As in the previous comparison, the difference in the growth rates of credit limits between these two groups is partly due to a lower starting credit limit of borrowers who end up opening a new card. However, most of this disparity in the growth rates is due to much larger resulting credit limits of borrowers with new cards, as can be seen in Table A5 in Appendix A.

since the overall credit limit includes the credit limit of the additional card. What is more surprising is that, in the case of emerging borrowers with a new card, a large portion of the overall increase in the credit limit comes from incumbent lenders. The growth in the incumbent cards' credit limit for these borrowers is 142.6%, which is roughly 63% of the overall (226.38%) increase. The corresponding number for the established borrowers with a new card is 5.75%, which is roughly 18% of the overall (31.32%) increase in the credit limit. For emerging borrowers, the growth in the incumbent cards' credit limit is far greater for borrowers who open a new card than for those who do not, 142.6% vs. 22.58%. The gap is much smaller for established borrowers, 5.75% vs. −2.55%.

We interpret these results as evidence of the information aggregation mechanism—that incumbent lenders of emerging borrowers interpret additional cards as positive news. There can be, of course, other channels that contribute to the observed facts. Throughout the paper, we argue that alternative explanations do not account for all of the facts that we present in subsections 2.3, 2.4, and 5.2. This by no means rules out the presence of the alternative channels, but rather suggests that the novel channel we explore is also important, especially for emerging borrowers.

One alternative explanation for the disproportional growth of the credit limit of (emerging) borrowers with new cards is the "demand" channel. By the demand channel we mean the conjecture that individuals with liquidity needs both seek new cards and approach incumbent lenders for credit-line increases, and that such liquidity-constrained borrowers would end up with larger credit limits both overall and on their incumbent cards. However, we would expect such individuals to have little *available*, or unused credit. This is not what we see in the data—as Table 4 documents, the growth rates of available credit follow exactly the same pattern as the growth rates of total credit limit.[21]

Table 4: Growth Rate of Aggregate Available Credit Limit from 2014 to 2015, %

	Emerging	Established
Total	49.31	2.47
Conditional on opening new card	221.62	32.54
Conditional on no new card	13.99	−3.16
Incumbent cards, cond. new card	142.38	6.27

Notes: The sample includes all borrowers with non-missing credit limits in 2014 and 2015 and with no more than five cards in 2015, or borrowers with non-missing credit limits in 2014 who have zero cards in 2015.

In order to further tease out the information-aggregation from the demand channel,

[21] While the growth rates are strikingly similar to those of the overall credit limits, the levels of unused credit are, of course, quite different, as can be seen from comparing Tables A5 and A7 in Appendix A.

we restrict the sample to borrowers for whom the demand channel is unlikely to be strong. First, we only look at borrowers whose aggregate balances after getting a new card do not exceed their aggregate credit limit before getting the new card. Second, we restrict the sample to borrowers whose aggregate utilization rate is below 50%. The results are reported in Tables A4 and A6 in Appendix A and are consistent with those in Tables 3 and 4.[22]

Moreover, Table A8 in Appendix A shows that borrowers who got a new card between 2014 and 2015 do not tend to have a substantially higher utilization in 2016 or 2017 than they did in 2014 or 2015.[23] This suggests that our results are not due to borrowers seeking additional credit limit in expectation of higher utilization in the future. All this evidence suggests that the demand channel is unlikely to be the primary driver of the observed differences.

Another potential explanation for why incumbent lenders increase the credit limits for borrowers who open a new card is competition among lenders for being the "top-of-the-wallet" card provider. That is, the incumbent lender wants the borrower to (continue to) use their card most frequently in order to collect transaction fees from retailers. Whether this "top-of-the-wallet" competition is more pronounced for emerging or established borrowers is theoretically ambiguous. On the one hand, if consumers' switching costs of moving from one credit card to another are increasing with the length of a credit relationship, then lenders might have an incentive to compete more fiercely for emerging borrowers to develop those relationships. On the other hand, established borrowers tend to have larger transactional balances and thus bring larger transaction fees implying that losing an established borrower may be more costly and may induce stronger top-of-the-wallet competition for established borrowers.

Empirically, we find evidence for both of these forces. Conditional on opening an additional credit card, emerging borrowers are indeed more likely to "switch" to the new card than established borrowers—balances on newly opened cards account for 28% of total balances for emerging borrowers but only for 16% for established ones. This finding is consistent with the idea that switching costs may be lower for emerging rather than established borrowers and could cause greater top-of-the-wallet competition for emerging borrowers. However, we also find that balances on newly opened cards are twice as large for established than for emerging borrowers—$1,032 and $511, respectively—suggesting that there may be greater competition for established rather than emerging borrowers.

[22]One, of course, has to be careful in interpreting results based on selection on ex-post outcomes. We view these results simply as additional supportive evidence for our mechanism.

[23]Winsorizing or trimming the data do not substantively affect these results.

Consistent with the above reasoning, the relative intensity of the top-of-the-wallet competition across the two groups appears ambiguous. In light of this ambiguity, we focus instead on the information-aggregation mechanism of credit-history building. We show that this mechanism is consistent with the empirical fact of "more dilution, lower default risk" that we document in subsection 5.2. The basic top-of-the-wallet mechanism, on the other hand, does not explain this additional empirical observation.

The results presented above are based on the total credit limit aggregated across cards. In subsection A.2 of Appendix A, we show results of the analysis at the individual card level. Specifically, we regress annual growth of card-level credit limits onto several control variables, while varying sample selection to narrow the potential mechanisms that may explain the results. The results, summarized in Table A2, are consistent with those presented in Table 3.

2.4 Lenders React to New Cards: Event Study

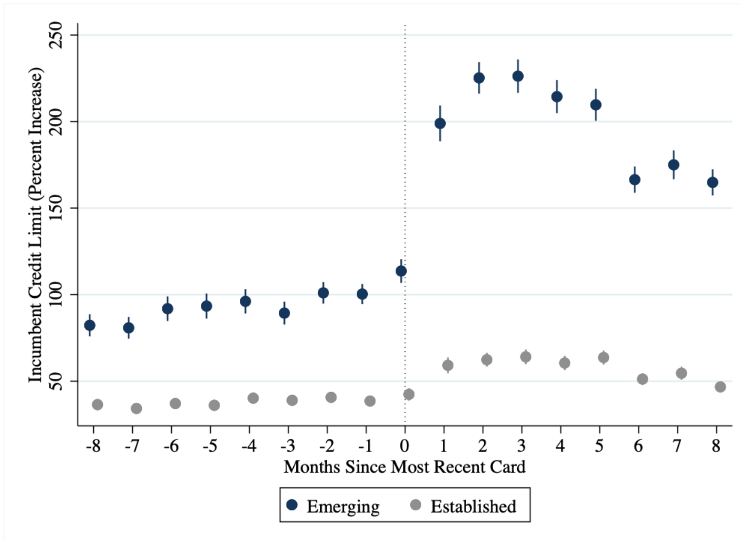
To lend further support to our hypothesis that lenders interpret the borrower's opening a new card as positive news and to limit alternative explanations, we zoom in on the exact timing of when a borrower opens a new card. Our data allow us to determine when a borrower opens a new account by examining the earliest month a card has a reported status indicator. We examine how credit limits on a borrower's existing cards change from 2014 to 2015 depending on *when* the borrower opens a new card. We treat this exercise as an event-study analysis to highlight the extent to which lenders respond to information on a borrower's credit report.

Specifically, we consider a sample of borrowers (emerging and established) who open a new credit card from eight months prior to the 2015 observation date (September 2015) to eight months after the 2015 observation date. That is, we look at cards opened between January 2015 and May 2016.[24] Figure 2 displays the average credit-limit growth (from 2014 to 2015) on incumbent cards for these borrowers by the month in which they open the card. We observe a stark discontinuity for emerging borrowers who open a new card before September 2015 rather than those who open a new card after. The timing of the jump in the incumbent's credit limit implies that the increase is not driven by observing the record of repayment, but rather by the fact of opening a new card.

Importantly, there does not appear to be any pre-trend in the credit growth for borrowers that were only one or two months away from opening a new card. This lack of

[24] We use information from our observation of a borrower's credit record in 2016 to determine if the borrower opened a card after the 2015 observation date.

Figure 2: Incumbent Lenders' Response



Notes: The figure plots the average card-level percent credit-limit growth (2015 limit minus 2014 limit divided by 2014 limit) for cards that are at least 12 months old separately by months since an individual opened their most recent card, where 0 represents individuals who opened a new card in September of 2015, -1 represents individuals who opened a card in October of 2015, and 1 represents individuals who opened a card in August of 2015. Missing card-level credit limits are set to the aggregate credit if an individual had one card. Timing of the credit card opening is determined by the earliest month implied by the account status indicator and constructed card linkages. Card-level credit limits are imputed as the difference between the reported total credit limit and all other card limits if only one card has missing credit-limit data.

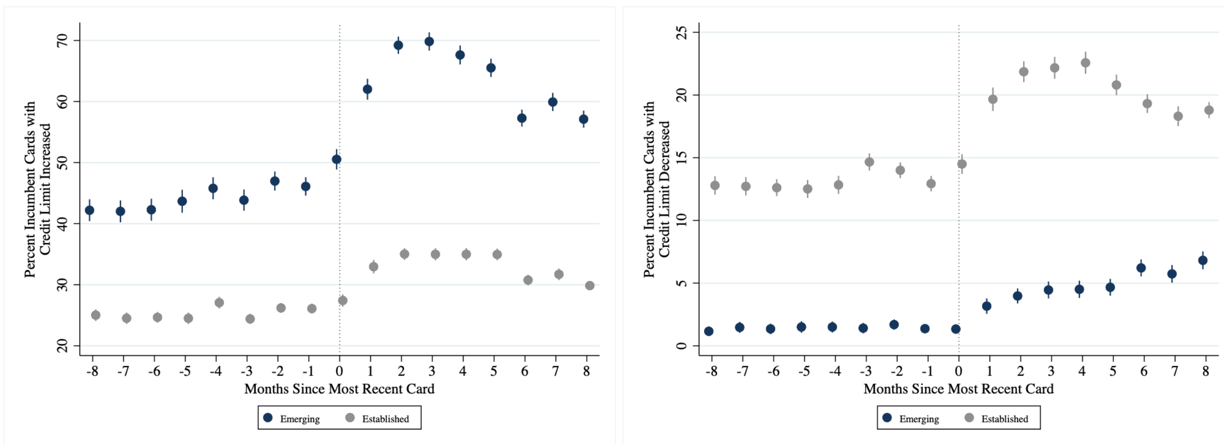
pre-trend strongly suggests that the increase in incumbent credit limits following a new card being opened is not primarily driven by a consumer's shock in demand for credit or her creditworthiness. Indeed, we would expect such a shock to act simultaneously on incumbent and new lenders, and thus occasionally the incumbent lender would move first, resulting in a clear pre-trend for the average borrower. Therefore, we interpret this result as suggestive evidence that lenders react to hard information, or "news," on a borrower's credit report.

While we do observe an overall increase in incumbent credit limits for established borrowers who open a card before September 2015 relative to those who open a new card after September 2015, the effect is muted relative to that observed for emerging borrowers. For example, established borrowers who opened a new card in July 2015, two months before the September observation month, had an average annual credit limit growth rate on incumbent cards of 62.42, while those who opened a new card in November 2015, two months after the observation month, had a growth rate of 40.67.[25] While non-trivial, this 22 percentage point increase in the credit limit growth rate is relatively small compared to the 124 percentage point increase observed within the emerging sample, which had

[25]The numbers used to construct the figure can be found in Table A9 in Appendix A.

growth rates of 225.24 and 101.05 for those who opened new cards in July and November, respectively.[26] These results are robust to a number of controls including the age of the card, the number of cards, prior utilization, types of credit, the credit score, and more—see Table A2 in Appendix A for details.

Figure 3: Incumbent Lenders' Response: Linear Probability Model



Notes: The left panel displays the share of incumbent cards (cards which existed in 2014) which had annual credit limit increases between 2014 and 2015 separately by months since an individual opened their most recent card. The right panel displays the share of incumbent cards which had annual credit limit decreases. The sample is restricted to individuals who opened a card +/- 8 months of September 2015. Missing card level credit limits are set to the aggregate credit if an individual had one card. Timing of the credit card opening is determined by the earliest month implied by the account status indicator and constructed card linkages.

We further explore this event study in Figure 3. The figure plots the share of incumbent cards that experience (i) an increase in the credit limit, or (ii) a decrease in the credit limit, as a function of the time when the borrower opens a new card.[27] The left panel confirms what we learned from Figure 2—incumbent lenders are much more likely to increase the credit limit for an emerging rather than for an established borrower after seeing a new card. On the other hand, these incumbent lenders are much more likely to *decrease* the credit limit for an established borrower in the month(s) immediately following a new card appearing on a credit record. This highlights the fact that lenders are indeed concerned with debt dilution, but those concerns are outweighed by the positive-news aspect of the new card for the emerging borrowers.

We further empirically explore the lenders' concern for debt dilution in subsection 5.2. We document that for emerging borrowers, a larger credit limit on a new card is associated with lower future delinquency rates. Such relationship is absent for estab-

[26] Proportional differences in growth rates are also stark: more than doubling for emerging borrowers, while the ratio of the two growth rates for established borrowers is about 1.5.

[27] Figure A1 in Appendix A also plots the share of closed incumbent cards as a function of the time when the borrower opens a new card.

lished borrowers. This observation is another piece of evidence that information about the borrower's creditworthiness may be contained in her getting a new card especially for emerging borrowers.

Our empirical analysis suggests that borrowing from multiple lenders plays a key role in the emerging borrowers' expansion of access to credit. We hypothesize that lenders perceive credit offered by other lenders as revealing new, favorable information about an emerging borrower. Next, we develop a novel framework that captures this idea in a parsimonious way. All borrowers in our model may be viewed as emerging. It is natural to assume that more information is commonly available about established borrowers, and hence the information-aggregation channel that we focus on is less important for them.[28] Instead, other channels that we discussed above, but abstract from in the model, are relatively more important for established borrowers. Thus, we would expect predictions of our model of emerging borrowers to be attenuated or possibly even reversed when it comes to established borrowers.

In Section 3, we present a simplified version of the model to illustrate the mechanism of credit-history building by aggregating information in the most straightforward way. In Section 4, we will present a more general model.

3 A Simple Model of Credit-History Building

3.1 Environment

There are two periods, I and II, and period I consists of two stages, 1 and 2. We study the interaction between a single borrower and multiple ($2 \times N$, $N \geqslant 2$) competing lenders.[29] The borrower has no endowment in period I.[30] Her endowment e in period II is stochastic, drawn from support $\{e_\ell, e_h\}$, where $0 < e_\ell < e_h$. The probability distribution over these endowment realizations depends on the borrower's state (or quality) $s \in \{g, b\}$. We will refer to borrowers with states g and b as good and bad borrowers, respectively. The borrower's quality is unobservable to anyone, including the borrower. We assume that $\Pr(e = e_h|s = g) = \gamma \in (0, 1)$, $\Pr(e = e_\ell|s = g) = 1 - \gamma$, and $\Pr(e = e_h|s = b) = 0$. The ex-ante probability that a borrower's quality is g (and the share of good borrowers in the

[28] In our model, the only reason that individuals borrow from multiple lenders is to aggregate information. Other reasons for having multiple credit cards, such as promotions, differentiated products, search frictions, etc., are presumably more important for established borrowers.

[29] We can equivalently assume that there are many borrowers.

[30] The assumption of zero endowment is for expositional simplicity only. All of our analysis and results extend if we assume that the borrower has a positive endowment in period I.

population) is $\alpha \in (0,1)$.

Each borrower derives utility from consumption in each of the two periods according to a per-period utility function $u\colon [0,+\infty) \to \mathbb{R}$. The function u is continuous, strictly increasing, and weakly concave. The borrower discounts period-II utility with the discount factor $\beta \geqslant 0$. There is no discounting across stages within period I. Lenders are risk neutral, have deep pockets, and discount period-II payoffs with the discount factor $\bar{q} = (1+\bar{r})^{-1}$, where $\bar{r}$ is the risk-free interest rate.

The only financial instrument available in the economy is a non-contingent defaultable bond payable in period II. Restricting attention to simple debt contracts forgoes the distinction between credit limits and credit balances but facilitates the transparency of the model. In Appendix B, we discuss an extension of our model to credit lines.

If the borrower defaults on a loan (fails to pay the full amount owed), she suffers a loss of fraction $\varphi \in (0,1]$ of her endowment. This cost of default is a dead-weight loss, as the lost portion of the borrower's endowment is destroyed and not transferred to the lenders.

At the beginning of period I, each lender receives a private signal, σ, about the borrower's quality.[31] The signals are binary, with support $\{A, B\}$. There are two equal-size classes of lenders, which differ only in the realization of the signal they receive.[32] Within each class, lenders observe the same signal, while signals across the two classes are conditionally independent. The signal is drawn from a distribution that depends on the borrower's quality: $\Pr(A|g) = 1$, $\Pr(B|g) = 0$, $\Pr(B|b) = (1+\rho)/2$, $\Pr(A|b) = (1-\rho)/2$, where $\rho \in [0,1]$. That is, observing signal B implies that the borrower is bad.

We now describe the interaction between the borrower and lenders as an extensive form game. In each stage of period I, lenders simultaneously offer contracts to the borrower. A contract is a pair (x, q), where x is the face value of the loan (equivalently, the amount of bonds the borrower sells) and q is the price. That is, a borrower who accepts a contract (x, q) from a given lender in a given stage of period I, receives qx from this lender in period I, and has a (defaultable) obligation to repay x to this lender in period II. We restrict the face value x to be no smaller than a minimal threshold $\underline{x} > 0$.[33]

[31] Our main results would not change if we assumed that the borrower learns the lenders' signals at the same time as the lenders. In fact, it would even simplify the characterization of off-equilibrium-path beliefs and strategies.

[32] The assumption of equal sizes of the two classes is only for concreteness. What is important is that there are at least two lenders in each class, and hence they will compete.

[33] The assumption that there is the smallest allowed loan size in the simple model is only needed in the case with the borrower's private information and $\beta > 0$ in subsection 3.2.3. We will need this assumption in the general model even under the simplifying assumption $\beta = 0$. It restricts lenders' cream-skimming deviations to smaller loans in equilibria with cross-subsidization.

Let $O_t = \{(x_t^k, q_t^k, k)\}_k$ denote the set of offered contracts together with the identities of lenders (k) offering these contracts in stage $t \in \{1, 2\}$. (A lender who does not offer a contract can be thought of as offering $(0, 0)$.) After observing the set of offered contracts in a given stage, the borrower accepts at most one contract in that stage.[34] That is, within a stage, contracts are exclusive.[35]

At the end of stage 1, all lenders observe the terms of the contract accepted by the borrower in stage 1 and the identity of the lender who offered it. We will refer to this public history as the *credit history* of the borrower. Formally, the (public) credit history is $h_2^P = (x_1, q_1, j_1)$ if a contract (x_1, q_1) from lender j_1 was accepted in stage 1, and $h_2^P = (0, 0, 0)$ if no contract was accepted.

Suppose the borrower accepts loans x_1 at q_1 in stage 1 and x_2 at q_2 in stage 2. The borrower's consumption in period I is then $q_1x_1 + q_2x_2$, and the total loan balance carried into period II is $X := x_1 + x_2$. In period II, after observing the realized endowment, e, the borrower chooses whether to repay or default on her debt obligations. Repaying anything less than X is equivalent to defaulting.[36] If the borrower defaults in period II, her consumption in that period is $(1 - \varphi)e$, and that of her lenders is 0. If the borrower repays X, she consumes $e - X$, and the lenders who lent in stages 1 and 2 consume x_1 and x_2, respectively. It follows immediately that the borrower will repay if and only if $e - X \geqslant (1 - \varphi)e$. This implies that the borrower's payoff is $\pi^B = u(q_1x_1 + q_2x_2) + \beta u(\max\{e - x_1 - x_2, (1 - \varphi)e\})$, and the payoff to a lender associated with a contract (x, q) that he offers and that the borrower accepts in (one of the stages of) period I is $\pi^L = -qx + \bar{q}x\mathbb{1}_{[\varphi e \geqslant X]}$. Appendix E presents the sequence of problems faced by each agent in the order implied by backward induction.

For notational convenience, we will refer to a lender who observes an A (B) signal realization as an A-lender (a B-lender). We will refer to a borrower for whom the pair of signal realizations for the two lender classes are A and B as an AB-borrower. Similarly, the AA-borrowers (BB-borrowers) are those for whom both classes of lenders observe an A (a B) signal realization. We will refer to the pair of signal realizations as the borrower's *type*. That is, the borrower's type can be AA, AB, or BB. Importantly, the borrower's type differs from her state, or quality s, which no one observes. Notice that whether a borrower is AA, AB, or BB is initially unknown to both the borrower and lenders. Whether

[34]We assume that if the borrower is indifferent between multiple offers, she accepts each of these offers with equal probability.

[35]The assumption that at most one contract can be accepted within a stage significantly simplifies the analysis. Intuitively, however, the incentive for a borrower to aggregate information is present even if she could accept multiple offers within a stage.

[36]Implicitly, this way of modeling default ensures that partial default is never optimal for the borrower.

borrowers or lenders may be able to infer this information depends on the strategies these agents choose.

We study pure-strategy Perfect Bayesian Equilibria (PBE) of the described game.

Definition 1 *A* Perfect Bayesian Equilibrium *consists of offer strategies for the lenders, acceptance strategies for the borrower, and posterior beliefs (for the lenders and the borrower) such that the lenders' and borrower's strategies are optimal and posterior beliefs satisfy Bayes' rule (where applicable).*

3.2 Equilibrium with Credit-History Building

In this subsection, we describe (a candidate) equilibrium with credit-history building. We then establish that it is indeed an equilibrium under a simple condition on the parameter values. To shorten the exposition, in the main text we only describe on-the-equilibrium-path strategies. The full descriptions of the equilibrium, including off-path strategies and beliefs, can be found in Appendix D.

Here is how credit-history building works. In equilibrium described below, only lenders with positive signals make offers in the first stage. A borrower who see offers from lenders in the first stage concludes that these lenders have positive signals about her. To transmit this information to other lenders, the borrower accept one of the offers. Lenders who see that the borrower accepted an offer conclude that this offer came from a lender with a positive signal, update their belief about the borrower's creditworthiness upwards, and offer better contract terms in the second stage.

3.2.1 Case with $\beta = 0$

We start by making a simplifying assumption that $\beta = 0$, i.e., borrowers simply maximize the amount of consumption they receive in the first period. As a result, there is no difference in valuation of contracts across the borrower's types, which simplifies our equilibrium analysis. Note that since $\beta = 0$, the equilibrium total loan sizes are in the set $\{\varphi e_\ell, \varphi e_h\}$. We will refer to the loans of these sizes as small and large, respectively. Since a borrower's default probability is the same for (total) loan sizes in each of the intervals, $(0, \varphi e_\ell]$, $(\varphi e_\ell, \varphi e_h]$, the corresponding equilibrium loan prices will be constant as well. Hence an impatient borrower will not choose an interior loan size but will prefer to be at the corner. Finally, we assume that the smallest loan size $\underline{x} \leqslant \varphi e_\ell$.

The (on-path) equilibrium strategies are as follows. In stage 1, B-lenders make no offers, and A-lenders offer a loan of size $\underline{x}$ at price $q_h^{AA} = \Pr(\text{repaying large loan}|AA)\bar{q} =$

$\Pr(e_h|AA)\bar{q}$. Only borrowers with two such offers (i.e., AA-borrowers) accept one. In stage 2, A-lenders whose offer was not accepted and who see an accepted offer that came from a lender from the other class, conclude that the borrower is AA. They offer a loan $\varphi e_h - \underline{x}$ (i.e., a top-up to a large loan) at price q_h^{AA}. An AA-borrower accepts such an offer. All other lenders (B-lenders and A-lenders who observe no accepted offer) offer a risk-free small loan φe_ℓ at $\bar{q}$. AB- and BB- borrowers accept it.

Consider who learns what when in this equilibrium. Because only A-lenders make offers in stage 1, the borrowers infer all the signals from seeing the stage-1 offers. A-lenders learn the signal of the other class of lenders at the end of stage 1. Indeed, if an A-lender sees an accepted offer from the other class of lenders, he concludes that the other class' signal is A. If he sees that no offer was accepted, he concludes that the other class' signal is B. The lender whose offer was accepted and lenders from the same class conclude that the other class' signal must be A, since only AA-borrowers accept stage-1 offers. Finally, B-lenders do not learn the other class' signal.

Borrowers understand this learning process, and take into account how their actions affect their credit histories. We refer to taking a stage-1 loan with the purpose of information aggregation as credit-history building. Formally, let $\lambda_{j,t}$ denote the probability that a lender from class j assigns to the borrower being of quality g at the beginning of stage t. We say that an equilibrium features *credit-history building* for a type $\omega \in \{AA, AB, BB\}$ if $\max_j \lambda_{j,2} > \max_j \lambda_{j,1}$. Under this definition, in the equilibrium described above, AA-borrowers build credit history. For AA-borrowers, all lenders update their beliefs from $\Pr(g|A)$ in stage 1 to $\Pr(g|AA)$ in stage 2. For AB-borrowers, A-lenders hold the most favorable beliefs in stage 1, but their beliefs worsen from stage 1 to stage 2. For BB-borrowers, lenders' beliefs do not change from stage 1 to stage 2.

Note that the reason why the prices of loans are actuarially fair in this equilibrium is that AB-borrowers choose not to accept the stage-1 offer. The reason for this is that $q_h^{AA} < \bar{q}$, which says that the AA-borrowers' actuarially fair price (for a risky loan that they receive in equilibrium) is lower than that of the AB-borrowers (given the safe loan they receive in equilibrium). Note further that if AA-borrowers were to reject the stage-1 loan, they would be perceived as AB-borrowers and offered a risk-free small loan in stage 2. Hence the condition for AA-borrowers to accept the stage-1 loan is $q_h^{AA} e_h \geqslant \bar{q} e_\ell$, which can be rewritten as

$$\Pr(e_h|AA)e_h \geqslant e_\ell, \tag{1}$$

where $\Pr(e_h|AA) = \gamma \Pr(g|AA) = \gamma\alpha(1+\rho)^2/[\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2]$.

For brevity, we refer to the described above (candidate) equilibrium simply as credit-history building.

Proposition 1 *Suppose that $\beta = 0$. Then credit-history building is an equilibrium if* (1) *holds.*

Condition (1) is intuitive—it says that an AA-borrower prefers to build credit history rather than getting a small loan in stage 2. If model parameters are such that condition (1) is violated, then there is different equilibrium, one without credit-history building. In this equilibrium, there is no borrowing in stage 1, and all borrowers get a small risk-free loan in stage 2.[37]

3.2.2 Case with $\beta > 0$

We now relax our assumption that borrowers are myopic and analyze the case with $\beta > 0$. We demonstrate that our mechanism of credit-history building is robust to this more realistic assumption. Assume that $\beta < \bar{q}$ so that there are gains from trade between the borrower and lenders. To simplify the analysis, we assume that the borrower is risk neutral, which ensures that the equilibrium total loan sizes are still in the set $\{\varphi e_\ell, \varphi e_h\}$.[38]

Below, we show that the result in Proposition 1 generalizes to the case with $\beta > 0$ with an appropriate modification of condition (1). To derive the new condition, we compare an AA-borrower's expected utility in the equilibrium with credit-history building, $\varphi e_h q_h^{AA} + \beta\{\Pr(e_h|AA)(1-\varphi)e_h + [1-\Pr(e_h|AA)](1-\varphi)e_\ell\}$, to her expected utility from taking no loan in stage 1 and taking a small risk-free loan in stage 2, $\varphi e_\ell \bar{q} + \beta\{\Pr(e_h|AA)(e_h - \varphi e_\ell) + [1 - \Pr(e_h|AA)](1-\varphi)e_\ell\}$. The former exceeds the later if and only if

$$\bar{q}\Pr(e_h|AA)e_h \geqslant \bar{q}e_\ell + \beta\Pr(e_h|AA)(e_h - e_\ell). \tag{2}$$

Condition (2) is the analog of condition (1). The last term on the right-hand side captures the gain in AA-borrower's utility in period 2 from repaying a smaller loan if she chooses to not build credit history. As in the $\beta = 0$ case, this condition means that an AA-borrower prefers to build credit history rather than taking a small risk-free loan in stage 2.

Proposition 2 *Let $\beta \in (0, \bar{q})$. Then credit-history building is an equilibrium if* (2) *holds.*

3.2.3 Private Information of the Borrower

So far we have assumed that the borrower does not have additional information about their ability to repay beyond that contained in the lenders' signals. We now argue that

[37] For brevity, we skip characterization of this equilibrium. We characterize an analog of this equilibrium in subsection 4.2.3 for the general model.

[38] With a strictly concave utility function, we would need to impose a restriction on the model parameter values to guarantee that the total loan sizes are in the set $\{\varphi e_\ell, \varphi e_h\}$.

our key insight about credit-history building is robust to the borrower having private information about their future income distribution. For this private information to matter, we must assume $\beta > 0$. (When $\beta = 0$, borrowers of all types value contracts in the same way and thus any such private information has no impact on the nature of equilibrium outcomes.) When $\beta > 0$ and borrowers have private information about their probability of repayment, different borrowers perceive the expected repayment costs of a given loan differently. Understanding these differences in perceived expected repayment costs, lenders may be able to use loan sizes and interest rates to induce different borrowers to self-select into different loans.

In order to address credit-history building in the presence of private information, we analyze an extension of our model in which there is a hidden privately observed state for the borrower (see Appendix D.3). This hidden state $z \in \{z_n, z_p\}$ is such that $\Pr(e = e_h | s = g, z = z_p) > \Pr(e = e_h | s = g, z = z_n)$ while $\Pr(e = e_\ell | s = b, z) = 1$ for any $z \in \{z_n, z_p\}$. That is, as before, a bad signal is fully revealing ($\Pr(g|AB, z) = \Pr(g|BB, z) = 0$ for any z) but among AA-borrowers, those with state z_p are less risky than those with state z_n ($\Pr(g|AA, z_p) > \Pr(g|AA, z_n) > 0$.) Let $\zeta = \Pr(z = z_n)$.

We establish the existence of an equilibrium with credit-history building analogous to that without private information but modified so that in stage 2 there is least-cost separation of AA-borrowers with states z_n and z_p. In this equilibrium, these borrowers receive different top-up loans. We demonstrate in Proposition 3 that under a condition similar to condition (2) and some additional restrictions on parameters (namely, ζ is small enough and e_h and e_ℓ are sufficiently far apart), credit-history building is still an equilibrium. Thus, our mechanism is robust to allowing for private information on the side of the borrower.

3.3 Debt Dilution

As in other models with borrowing from multiple lenders, our equilibria suffer from debt dilution—taking an additional loan decreases the probability of repayment of the initial loan. In the equilibrium with credit-history building, only AA-borrowers take out an additional loan in stage 2. Conditional on being an AA-borrower, an additional loan decreases the probability of repayment. This is the standard "dilution effect." Notice that a large, risky loan is taken by better quality borrowers in equilibrium, while a small, safe loan is taken by worse quality borrowers. This is a "selection effect" associated with credit-history building. We will show that in a more general model that we analyze in Section 4 this selection effect leads to a new testable implication about debt dilution, which holds in the data.

3.4 Building a Credit History vs. Improving a Credit Score

Our mechanism highlights an important distinction between credit-history building and improving one's credit score. Since the purpose of a credit score is to proxy a borrower's probability of repayment (see, e.g., Fair Isaac Corporation, 2022), and information aggregation leads to larger and hence riskier loans for more creditworthy borrowers, the credit-history building that emerges in equilibrium results in *lower* credit scores for these borrowers. Taking on an early loan communicates positive information to other lenders, lowering the posterior probability of default on a given loan size. But, since information aggregation induces borrowers to take on a *larger* loan, the resulting probability of default is increased (relative to those borrowers who do not take on early loans).

A straightforward way to see this in the model is to consider the equilibrium in subsection 3.2.1. The borrower's beginning-of-stage-1 credit score (i.e., the ex-ante, or the uninformed probability of repayment) is $\Pr(e_h|AA)\Pr(AA) + 1 \times [1 - \Pr(AA)]$. The end-of-stage-1 credit score (i.e., the posterior probability of repayment) of an AA-borrower is $\Pr(e_h|AA)$. It is lower than the corresponding credit score of an AB- or a BB-borrower (whose probability of repayment is 1), and lower than the ex-ante credit score. That is, while an AA-borrower chooses to build a credit history to improve her loan terms, she lowers her credit score while doing so.

This observation is more than just a theoretical oddity. Our empirical findings in Section 5 are consistent with this phenomenon—opening an additional card is associated with a greater probability of default, for both emerging and established borrowers. At the same time, as we documented in Section 2, incumbent lenders respond positively to a borrower opening an additional credit card (more so for emerging than for established borrowers).

4 General Model

In this section, we consider a generalization of the model we presented in the previous section. This model features three endowment levels and a more general signal structure. The main goal is to add an intensive margin of dilution, which will lead to the model's testable implication. To get this, we need an additional endowment level so that there is dilution to two different loan sizes in the second stage, and a more general signal structure so that a negative signal does not fully reveal the borrower's bad quality. Additionally, the general model allows us to highlight that credit-history building can be costly due to excessive borrowing and to analyze welfare implications.

4.1 Environment

The environment is similar to the one presented in subsection 3.1 with the following modifications. First, the borrower's endowment e in period II is drawn from the set $\{e_\ell, e_m, e_h\}$, where $0 < e_\ell < e_m < e_h$, so there is an additional endowment level, e_m. We assume that the endowment distribution of good borrowers first-order stochastically dominates that of bad borrowers. Second, we dispose of our earlier assumption of the extreme signal structure $\Pr(A|g) = 1$ and instead assume that $\Pr(A|g) \in [0, 1]$.

To simplify our analysis, as in subsection 3.2.1, we assume that $\beta = 0$. We also assume that the borrower is risk neutral, which is not particularly restrictive given $\beta = 0$. This assumption simplifies analysis of situations when in equilibrium no action is taken in the first stage, and so the borrower needs to form expectations about her quality in the event of a lender's deviation in the first stage. It also eliminates redistributional consideration when evaluating ex-ante welfare.

We also assume that the endowment distribution is such that bad borrowers only receive low or medium endowments, while good borrowers only receive medium or high endowments. Moreover, the probability of receiving the medium endowment is the same for both good and bad borrowers. Formally, $\Pr(e_\ell|b) = \Pr(e_h|g) = \delta$, $\Pr(e_m|b) = \Pr(e_m|g) = 1 - \delta$, $\Pr(e_h|b) = \Pr(e_\ell|g) = 0$. These assumptions on the endowment distribution are not crucial for our analysis and are only made to reduce the number of parameters and to simplify the algebra.

Finally, we assume that the distribution of signals is symmetric so that $\Pr(A|g) = \Pr(B|b) = (1 + \rho)/2$. We refer to $\rho \in [0, 1]$ as the *precision* of the signal. Again, this assumption is not crucial and is made to simplify algebra.

As in the simple model, since $\beta = 0$, the equilibrium total loan sizes are in the set $\{\varphi e_\ell, \varphi e_m, \varphi e_h\}$. We will refer to these loans as small, medium, and large, respectively. Finally, for simplicity, we set $\underline{x} = \varphi e_\ell$.

4.2 Equilibria

As in Section 3, we focus on pure-strategy PBE. Among pure-strategy PBE, we select the one(s) preferred by AA-borrowers. This equilibrium selection is not intended to focus on AA-borrowers per se; instead, it is intended to rule out equilibrium outcomes supported by "unintuitive" beliefs in the spirit of Cho and Kreps (1987). Specifically, it rules out equilibria with more than the minimally needed level of cross-subsidization in stage 1.[39]

This selection criterion generically picks a unique equilibrium outcome for a given set of parameter values. This equilibrium changes as parameters vary. In what follows, we analyze three specific equilibria—two with credit-history building and one without.[40] The first equilibrium is the analog of the credit-history building equilibrium in the simple model. The second equilibrium is key for deriving our testable implication in Section 5. The third equilibrium is an important benchmark for analyzing welfare implications of the model. In Appendix C we present other equilibria and explore numerically how the selected equilibrium changes depending on parameter values.

To better understand the role of credit-history building and the threat of dilution that comes with it, it is instructive to first consider two special cases. In particular, suppose that the informativeness of the lenders' signals takes one of the two extreme values, $\rho = 0$ and $\rho = 1$. If $\rho = 0$, the signals are pure noise, and the posterior equals to the prior. If $\rho = 1$, the signal perfectly reveals the borrower's quality. Importantly, in both of these cases lenders in different classes will have the same belief (and information) about the borrower's creditworthiness. As a result, there is no need to aggregate information, and there is no credit-history building in equilibrium. Note that stage-1 borrowing is potentially subject to dilution, while stage-2 borrowing is not because there is no further opportunity to borrow. Since there is no benefit to borrowing early, without loss of generality, all borrowing happens in stage 2, and there is no debt dilution. In both $\rho = 0$ and $\rho = 1$ cases, given the common posterior about the borrower's creditworthiness, the borrower gets an actuarily-fair priced loan that maximizes her expected utility.[41]

The situation is quite different when the signal precision is interior, $\rho \in (0, 1)$. In this case, the beliefs of lenders are affected by learning the other lenders' signals. To aggregate the dispersed information, the borrower might choose to build credit history by borrowing early, and thus exposing an early loan to dilution in stage 2.

[39] Sustaining more cross-subsidization from AA- to AB-borrowers in stage 1 requires off-equilibrium-path beliefs that a smaller stage-1 loan would be more likely to be accepted by an AB- rather than an AA-borrower, whereas AAs' incentives to accept such a deviation are necessarily stronger. This is the sense in which our equilibrium selection is in the spirit or the intuitive criterion. (The intuitive criterion of Cho and Kreps, 1987 does not directly apply in our environment because of the richness of the strategic interactions that come *after* the signaling takes place in our model.)

[40] As in the simple model, to shorten the exposition, in the main text we only describe on-path strategies. The full descriptions of all equilibria, including off-path strategies and beliefs, can be found in online Appendix F.

[41] Of course, the loan sizes and prices differ under $\rho = 0$ and $\rho = 1$—see our discussion in Appendix C.

4.2.1 Equilibrium with credit-history building, ℓmh outcome, and no cross-subsidization

We now consider an equilibrium that features credit-history building and results in the ℓmh outcome, that is, small, medium and large loans to BB-, AB- and AA-borrowers, respectively. This equilibrium is an analog of the equilibrium we analyzed in subsection 3.2.1. The (on-path) equilibrium strategies are as follows. In stage 1, B-lenders make no offers, and A-lenders offer a loan of size φe_ℓ at price

$$q_h^{AA} = \Pr(\text{repaying large loan}|AA)\bar{q} = \Pr(e = e_h|AA)\bar{q}. \tag{3}$$

Only borrowers with two such offers (i.e., AA-borrowers) accept one. In stage 2, A-lenders whose offer was not accepted and who see that the accepted offer came from a lender from the other class, conclude that the borrower is AA. They offer a loan $\varphi(e_h - e_\ell)$ (i.e., a top-up to a large loan) at price q_h^{AA}. An AA-borrower accepts such an offer. A-lenders who do not see an accepted loan offer φe_m at price q_m^{AB} defined as

$$q_m^{AB} = \Pr(\text{repaying medium loan}|AB)\bar{q} = \Pr(e \in \{e_m, e_h\}|AB)\bar{q}. \tag{4}$$

An AB-borrower accepts such an offer. Finally, B-lenders (who see that no offer was accepted in the first stage) offer a risk-free small loan φe_ℓ at $\bar{q}$. A BB-borrower accepts it.

In this equilibrium, AA-borrowers build a credit history: beliefs of A-lenders about AAs' creditworthiness improve from stage 1 to stage 2. As a result, AA-borrowers can get a better price for any given size loan (compared to if they did not accept the stage-1 offer and would get mistaken for AB-borrowers). In the equilibrium above, when the AA-borrowers face these improved interest rates, they choose to take on more credit.

Importantly, credit-history building may come at a cost. The potential cost of credit-history building is *excessive borrowing*, meaning that the resulting loans of AA-borrowers are larger than what they would have been under symmetric information, where all signals are public information.[42] In this equilibrium, excessive borrowing happens when the symmetric-information outcome has AA-borrowers ending up with a medium-size loan. Formally, excessive borrowing is captured by the following two conditions:

$$q_h^{AA} e_h < q_m^{AA} e_m, \tag{5}$$

$$q_h^{AA}(e_h - e_\ell) > q_m^{AA}(e_m - e_\ell), \tag{6}$$

where $q_m^{AA} = \Pr(\text{repaying medium loan}|AA)\bar{q} = \Pr(e \in \{e_m, e_h\}|AA)\bar{q}$. Condition (5)

[42] We define excessive borrowing in terms of the face value of the loan X.

guarantees that AA-borrowers choose the medium-size loan under symmetric information. Condition (6) states that, after taking on a small loan in stage 1, AA-borrowers prefer topping up to the large loan, rather than the medium one.[43] Conditions (5)-(6) can be rewritten as

$$\frac{e_m - e_\ell}{e_h - e_\ell} < \frac{q_h^{AA}}{q_m^{AA}} < \frac{e_m}{e_h}, \tag{7}$$

where q_h^{AA} and q_m^{AA} are independent of endowments. As e_ℓ becomes close to e_m, the first inequality in (7) is more likely to be satisfied. That is, as e_ℓ moves close to e_m, the top-up to a medium loan in the second stage becomes smaller, and the AA-borrowers' temptation to top up to a larger loan becomes stronger.

The equilibrium we have described may or may not feature excessive borrowing depending on parameter values.[44] Notice that when there is no excessive borrowing, any borrower receives the same size loan and at the same actuarially fair price as they would under symmetric information. Thus, credit-history building is costless in the case of no excessive borrowing.

Finally, note that the reason why the prices of loans are actuarially fair in this equilibrium is that AB-borrowers choose not to accept the stage-1 offer. The condition that ensures this behavior is

$$q_h^{AA} \leqslant q_m^{AB}. \tag{8}$$

Condition (8) says that the AA-borrowers' actuarially fair price is lower (or the interest rate is higher) than that of the AB-borrowers (given the loans they receive in equilibrium). In this equilibrium, AA-borrowers do not cross-subsidize AB-borrowers. Next, we consider an equilibrium that again results in the ℓmh outcome, but features cross-subsidization, and hence different loan prices compared to this equilibrium. Model parameters determine whether cross-subsidization occurs in equilibrium or not.

4.2.2 Equilibrium with credit-history building, ℓmh outcome, and cross-subsidization

Next, we consider an equilibrium that results in the ℓmh outcome, but both AA- and AB-borrowers accept the same stage-1 offer. This equilibrium will be key in deriving our testable implication in Section 5.

Symmetric pure-strategy equilibria where A-lenders from different classes make the same offers in stage 1 may not exist due to cream-skimming (market-stealing) incen-

[43] Note that by overborrowing, AA-borrowers reduce their probability of repayment ("credit score") compared to the symmetric-information benchmark by $\left(q_m^{AA} - q_h^{AA}\right)/\bar{q} = 1 - \delta$. Whether excessive borrowing occurs depends on the signal precision ρ, but the difference in the repayment probabilities does not.

[44] Note that due to condition (1), excessive borrowing never occurs in the simple model.

tives.[45] Instead, we construct an asymmetric equilibrium, in which A-lenders from different classes make different offers (within a class, lenders make the same offer). In particular, lenders from the first (second) class will offer a more (less) favorable price whenever they get an A signal. For the ease of exposition, we will refer to the first class as "green" or G for "generous" and to the second class as "fuchsia" or F for "frugal."

The (on-path) equilibrium strategies are as follows. In stage 1, B-lenders from either class make no offers. Green A-lenders offer φe_ℓ at price q^A, defined as

$$q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}. \tag{9}$$

Fuchsia A-lenders offer φe_ℓ at price q_m^{AB}. All borrowers who receive an offer accept one. If they receive offers from both classes of lenders, they accept one from a green lender. In stage 2, fuchsia A-lenders, whose offers were not accepted and who see the accepted offer made by a green lender, conclude that the borrower is AA. They offer a loan $\varphi(e_h - e_\ell)$ (i.e., top up to a large loan) at price q_h^{AA}. An AA-borrower accepts such an offer. A-lenders whose offer was accepted (or whose offer was not accepted, but the accepted offer came from a lender of the same class), or B-lenders who observed that an offer was accepted, offer $\varphi(e_m - e_\ell)$ at price q_m^{AB}. An AB-borrower accepts such an offer from one of those lenders. (Notice that A-lenders making such an offer correctly predict that only an AB-borrower would accept their offer.) Finally, B-lenders who see that no offer was accepted conclude that this is a BB-borrower and offer her a risk-free small loan φe_ℓ at $\bar{q}$. A BB-borrower accepts such an offer.

Let us first comment on the price of the stage-1 loan given in (9). Since both AA- and AB-borrowers accept this loan, the price reflects the default risk of both of these borrowers. That is, AA-borrowers cross-subsidize AB-borrowers. In addition, the small stage-1 loan will be diluted in stage 2 to either a medium loan (for AB-borrowers) or a large loan (for AA-borrowers). Hence, the price q^A of the stage-1 loan is a weighted average of the price of a large loan that only AA-borrowers accept and a medium loan that

[45] Consider a candidate symmetric pure-strategy equilibrium, in which green and fuchsia A-lenders make identical offers to borrowers. Such an offer would be accepted with probability $\frac{1}{K}$ by an AB-borrower, but only with probability $\frac{1}{2K}$ by an AA-borrower, reflecting the "winner's curse." An A-lender from either class has an incentive to offer a slightly better price than the conjectured equilibrium price in an effort to capture all of the market, thus improving the average quality of the pool. Should the market correctly interpret such an out-of-equilibrium offer as having come from an A-lender, this deviation would be profitable as it would result in an improved expected quality of the borrower. For this deviation to be unprofitable, it must fail to attract AA-borrowers. The only way to prevent AA-borrowers from accepting such a deviation offer is for such a loan to be interpreted (by the other class of lenders) as offered by a B-lender (with sufficient probability). However, a B-lender would always find such an offer unprofitable. Since we find the beliefs that could possibly support such a symmetric equilibrium unintuitive, we instead focus on the asymmetric equilibrium described below.

only AB-borrowers accept. The condition for cross-subsidization that ensures that AB-borrowers are willing to accept the same first-stage loan as AA-borrowers is the reverse of (8) in the previous subsection:

$$q_h^{AA} > q_m^{AB}. \tag{10}$$

Condition (10) restricts the set of parameter values such that cross-subsidization can happen in an ℓmh equilibrium. Since q_h^{AA} is increasing in the signal precision, ρ, and since q_m^{AB} does not depend on ρ (because the signals are symmetric), cross-subsidization can only occur in such an equilibrium when ρ is sufficiently high.[46] For large values of ρ—that is, when signals are precise—the default risk of AA-borrowers on a large loan is small, and so the interest rate on the stage-1 loan is low. The low interest rate makes the loan attractive to AB-borrowers.

Note that, as in the simple model, only AA-borrowers build a credit history. AB-borrowers do not accept the stage-1 offer in the hopes of obtaining better terms of credit in stage 2, but instead to free-ride on a better cross-subsidized price.

The equilibrium described above may or may not feature excessive borrowing depending on whether in the symmetric-information benchmark the AA-borrowers end up with a medium or a large loan. In online Appendix F, we provide the conditions for the model parameters so that one or the other scenario occurs.

4.2.3 Equilibrium without Credit-History Building

We now describe an equilibrium that features no information aggregation and results in the ℓmm outcome. In this equilibrium, no lender makes an offer in stage 1. In stage 2, green A-lenders offer a medium loan φe_m at $q_m^A = \Pr(AA|A)q_m^{AA} + \Pr(AB|A)q_m^{AB}$. All borrowers with such an offer accept it. Fuchsia A-lenders offer φe_m at q_m^{AB}. AB-borrowers with such an offer accept it. B-lenders offer a small loan φe_ℓ and $\bar{q}$. All borrowers with only such offers (i.e., BB-borrowers) accept one.[47]

Notice that in this equilibrium AA-borrowers cross-subsidize AB-borrowers on the whole (medium-size) loan rather than only on the small loan as in the equilibrium in subsection 4.2.2 or not at all in the equilibrium in subsection 4.2.1. Credit-history building allows AA-borrowers to limit cross-subsidization and get better loan prices. Furthermore, lenders' improved beliefs allow AA-borrowers to potentially borrow more than they would be able to if they did not build credit history.

[46] In Appendix C we illustrate other equilibrium outcomes where cross-subsidization occurs for low values of precision.

[47] The assumption of $\beta = 0$ is crucial for existence of such equilibrium as it eliminates lenders' ability to cream-skim in stage 2.

However, the ability to borrow in the first stage is a double-edged sword. It is beneficial if there is no excessive borrowing but may be detrimental otherwise. Notice that equilibria without credit-history building cannot have excessive borrowing, because all borrowing happens at once and hence there is neither dilution nor the problem of lack of commitment to future actions.

When a PBE with credit-history building results in excessive borrowing, AA-borrowers may prefer not to build a credit history. To illustrate this point intuititely, consider such a case with ρ very close to 1. (To have excessive borrowing at ρ close to 1, the symmetric-information outcome must be ℓmm.) Since the fraction of AB-borrowers (the probability of the pair of signals being AB) goes to 0 as ρ approaches 1, the no-information-aggregation outcome approaches that in the symmetric-information environment. In contrast, the equilibrium with credit-history building still features excessive borrowing, the cost of which does not converge to 0 as ρ tends to 1. Thus, for ρ close enough to 1, AA-borrowers prefer the equilibrium outcome without information aggregation.[48]

4.3 Welfare Implications

Our equilibrium selection picks credit-history building on a larger set of parameter values than what is desirable from the borrowers' ex-ante perspective (before the signals are realized).[49] The reason is that equilibrium without credit-history building features more cross-subsidization; the AA-borrowers dislike cross-subsidization, while the planner does not care about it.[50] Sometimes, borrowers would ex ante prefer (to commit) not to build credit history but ex post AA-borrowers find it in their interest to do so.

Thus, our model suggests an important welfare implication: publicly recording borrowers' credit histories is not always desirable from social welfare perspective. Availability of credit records allows lenders to tailor loans based on more precise information, but it may lead to excessive borrowing by AA-borrowers. When the cost outweighs the benefits, availability of credit records reduces ex-ante social welfare. We illustrate this welfare result using a numerical example in Appendix C.

Our model also allows us to think about the following related question: In the presence of credit records, does more precise information—e.g., arising from an improvement in lenders' statistical models—make borrowers better off? In Appendix C we demon-

[48]In Appendix C, we show that AA-borrower may prefer not to build a credit history for other parameter values as well using a numerical example.

[49]Lenders in our model break even, so the social welfare is equal to the borrower's expected utility.

[50]With risk-neutral borrowers, cross-subsidization does not matter for the ex-ante utility. If the borrowers were risk averse, cross-subsidization would create insurance and increase the ex-ante utility. This would further strengthen the result that AAs build credit history too often from the ex-ante perspective.

strate that the ex-ante welfare can be non-monotone in the quality of information. While generally welfare rises with the signal quality, it can drop discontinuously in some cases. The reason the welfare can drop is, once again, excessive borrowing. As signals become more precise, AA-borrowers get better terms on any given-size loan in the second stage. Sometimes these better terms lead them to overborrow, which results in a drop in their ex-ante utility.

Thus, our simple model yields surprisingly complex predictions regarding the desirability of public credit records. The possible welfare loss is due to debt dilution. Interestingly, in our environment public credit records do not prevent but instead encourage borrowing from multiple lenders that leads to debt dilution.

It is important to keep in mind, however, that our welfare analysis ignores the role of credit records as histories of debt *repayment*. As such, credit histories help alleviate adverse selection (by aggregating information about repayment) and moral hazard (by incentivizing borrowers to make repayments). Hence our welfare analysis only highlights the new consideration of aggregating information by taking loans, the force that is particularly important for emerging borrowers.

5 Testable Implications

In this section, we describe the model's novel prediction of "more dilution, lower default risk" and show that it is borne out in the data.

5.1 More Dilution, Lower Default Risk

As in other models with borrowing from multiple lenders, our equilibria suffer from debt dilution—taking an additional loan decreases the probability of repayment of the initial loan. However, our model generates a counterintuitive prediction that we refer to as "more dilution, lower default risk:" when the incumbent lender faces uncertainty about how much his early loan will be diluted, he is actually *more* likely to be repaid when the borrower takes a *larger* additional loan from another lender.

To see this, consider the equilibrium with the ℓmh outcome and cross-subsidization analyzed in subsection 4.2.2. Notice that in this equilibrium, there is uncertainty for the stage-1 lender about how much his loan will be diluted in stage 2. If the borrower turns out to be an AA-borrower, the stage-1 loan will be diluted to a large loan, and if the borrower turns out to be an AB-borrower, the loan will be diluted to a medium loan. Although the lender earns zero profit ex ante, in which of these two scenarios is he better off ex post? In other words, in which of the two cases is the probability of being repaid higher? The answer immediately follows from equation (10) and the definitions of q_h^{AA} and q_m^{AB} (equations (3) and (4))—in this equilibrium, an AA-borrower is more likely to repay a large loan than an AB-borrower to repay a medium loan. That is, the incumbent lender is more likely to be repaid if his initial loan is diluted by more.

Our result offers a subtle perspective on the conventional wisdom that dilution increases the subsequent probability of default. While our model is consistent with this standard "dilution effect" in that borrowers who take on subsequent loans have a higher risk of default on average, among those who take on such loans, it is the less risky/better quality borrowers who take on larger loans—a "selection effect". This selection effect dominates the dilution effect in the considered equilibrium. It is important to note that information aggregation is key for the result that more dilution implies a lower probability of default: a larger top-up loan conveys positive information of the diluting lender.

5.2 Model Validation

Our model attributes a significant degree of strategic behavior to lenders: they respond to credit trades made by other lenders that are accepted by borrowers. A priori, it is not obvious that lenders pay attention or respond to a borrower's additional loans. Our preliminary empirical findings presented in Section 2 suggest that lenders not only react to such new trades, but do so in a rather, perhaps, a priori counter-intuitive fashion—they tend to extend their own credit line in response to observing an "entrant." This observation is strongly supportive of the main signaling mechanism we are putting forward.

Another way to directly assess the validity of our model is to test the prediction of "more dilution, lower default risk" described in the previous subsection. We will do so by considering delinquency rates of borrowers following expansions of access to credit. Consider first the impact of additional credit on subsequent delinquency rates using the following Probit specification:

$$Probit(Delinquency_{i,2016}) = \beta_0 + \beta_1 NewCL_{i,2015} + \beta_2 NewCard_{i,2015} + \beta_3 X_{i,2014-15} + \varepsilon_i.$$

In this specification, $Delinquency_{i,2016}$ is the indicator of any credit card trade being more than 90 days past due in 2016. $NewCL_{i,2015}$ is the credit limit on all cards opened by individual i between 2014 and 2015 divided by his/her total credit limit in 2014. $NewCard_{i,2015}$ is a dummy variable that equals one if individual i opened a new card between 2014 and 2015 and equals zero otherwise. $X_{i,2014-15}$ is a set of control variables in 2014 and/or changes from 2014 to 2015. These variables reflect the financial state of the borrower prior to the expansion of credit. The sample in the regression includes only borrowers who had a credit card in 2014.

We run this regression separately for emerging and established borrowers. While we do not make an explicit distinction between emerging and established borrowers in the model, we expect our mechanism to be more pronounced for emerging borrowers: the informational content of an additional credit line is larger for emerging borrowers than for established borrowers, whose credit records contain a wealth of other information.

Table 5: New Card and Future Delinquency: Probit

	Emerging		Established	
	(1)	(2)	(3)	(4)
New card limit '14-'15 (share '14 lim)	-0.0010***	-0.0011***	0.0038***	-0.0004
	(0.0001)	(0.0001)	(0.0003)	(0.0003)
Opened new card '14-'15 (0/1)	0.0438***	0.0354***	0.0117***	0.0095***
	(0.0013)	(0.0012)	(0.0011)	(0.0010)
Vantage score '14		-0.0006***		-0.0008***
		(0.0000)		(0.0000)
N	206,951	206,951	285,529	285,529
Sample avg. delinquency rate	0.052	0.052	0.060	0.060
Pseudo R^2	0.0164	0.1104	0.0026	0.2280

Notes: The table displays marginal effects from a probit regression of a dummy for any card trade more than 90 days past due in 2016 onto the indicated row variables. The sample is conditional on having an open bank card in 2014 and 2016. The new card limit corresponds to the total card-level credit limit on all cards opened between 2014 and 2015. The reported average marginal effects of the new card limit reflect the average marginal effects of an increase in a borrower's new card limit computed only among the population of borrowers who opened a new card from 2014 to 2015.

The results of the regression (marginal effects) are reported in Table 5.[51] The left (right) panel contains the results for emerging (established) borrowers. Specification 1 presented in columns (1) and (3) does not contain additional control variables $X_{i,2014-15}$. Specification 2 presented in columns (2) and (4) contains only one additional control variable $X_{i,2014-15}$—the Vantage score in 2014. The idea is that the Vantage score should summarize financial information relevant for predicting delinquency. As a robustness check, instead of the Vantage score, Table A10 in Appendix A contains several other control variables: the total limit on credit cards in 2014, the total credit limit on all trades in 2014, the increase in the credit limit on the old card from 2014 to 2015, and several dummy variables.[52] (See also Table A11 for the summary statistics corresponding to Tables 5 and

[51] We have also conducted a similar analysis using charge-off rates as an alternative measure of loans' ex-post performance. The results of that OLS regression are consistent with the fundings presented here and are available upon request.

[52] Notice that while the coefficients of interest—β_1 and β_2—are very similar for these additional specifications in Table A10 in Appendix A.3, the pseudo R^2 is a lot higher for the specification with the 2014 Vantage score than for specifications with many controls. This tells us that indeed the Vantage score serves

A10.)

The key takeaway from these regressions is that, among emerging borrowers, a greater expansion of credit is associated with lower delinquency rates, just like our theory predicts. This is reflected in negative significant coefficients in the first row in columns (1) and (2). The coefficient -0.0011 implies that an increase in a borrower's new card limit equal in size to their total 2014 credit limit is associated with a 0.11 percentage point decrease in the 2016 delinquency rate on average. Notably, this negative relationship is absent for established borrowers, as reflected in a non-significant coefficient in the first row in column (4) (or a positive coefficient in column (3)). This is consistent with the selection effect being more pronounced for emerging borrowers than for established borrowers.

The striking observation that more dilution is associated with a lower default risk (when it comes to emerging borrowers) coexists with the conventional observation that dilution increases the default risk. We observe that borrowers who do not open a new card in 2015 have lower delinquency rates in 2016 than those who opened a new card, as illustrated in the second row of Table 5: the coefficient β_2 is positive and significant.[53] In the context of our model, the borrowers who do not open a new card correspond to borrowers who only borrow in stage 2. In equilibria described in subsections 4.2.1 and 4.2.2, the borrowers who only borrow once have lower default rates than those who borrow from multiple lenders. These are either BB-borrowers or AB-borrowers who declined a stage-1 offer (in the equilibrium without cross-subsidization). Since BB-borrowers only receive safe loans, they never default in equilibrium. AB-borrowers only decline a stage-1 offer when their equilibrium default rate is lower than that of AA-borrowers borrowing from multiple lenders. Thus, the seemingly conflicting observations—that dilution increases the default risk, but more dilution lowers the default risk—both emerge from our model.

Note that there is a key difference between our environment and existing models that analyze debt dilution (e.g., Aguiar et al., 2019, Bizer and DeMarzo, 1992, Chatterjee and Eyigungor, 2012, Hatchondo et al., 2016). In our model, there is a way to avoid debt dilution entirely, since loans made in stage 2 are not subject to dilution. And in the absence of the signaling motive (i.e., credit-history building), there would be no first-stage loans (and

as a sufficient statistic that incorporates the relevant information about the borrower's future probability of delinquency. Interestingly, while the pseudo R^2's for emerging and established borrowers are similar under specifications with many controls, the pseudo R^2 in the specification with the Vantage score is much lower for emerging borrowers than for established borrowers. This suggests that the Vantage score has less predictive power when it comes to predicting delinquency for emerging borrowers. This result is intuitive—there is less information available and a shorter credit history for emerging borrowers.

[53] Figure A2 in Appendix A.3 offers a visual comparison between the delinquency rates for borrowers who opened a new card—varying its credit limit—and those who did not open a new card.

hence, no dilution) in our environment. The only reason borrowers expose themselves to (sometimes) costly dilution is the desire to aggregate information across heterogeneously informed lenders.

6 Conclusion

We have explored, both empirically and theoretically, how emerging borrowers build their credit histories. We highlight the importance of taking on loans as a way for borrowers to aggregate information across potential lenders. This credit-history building mechanism emphasizes the role consumers' credit choices—and not just repayment behavior—play in determining their access to credit. Using novel data, we have documented that this mechanism is particularly important for understanding access to credit for emerging borrowers—those borrowers who have little to no existing credit history.

We have offered a parsimonious model capturing the idea of credit-history building as information aggregation. The model highlights trade-offs associated with credit-history building, both at the individual and at the societal level. One particularly striking model implication concerns debt dilution. The standard mechanism, which is present in our model, implies that when a borrower of a given quality increases her overall loan size, she also increases her probability of default. On the other hand, the novel information-aggregation channel present in our model suggests that larger loans are chosen by higher quality (or less risky) borrowers. Hence, in our model, a lender prefers to see his borrower taking on a larger, rather than a smaller, additional loan from a competing lender. Strikingly, our empirical evidence on the loan choices and default behavior of (emerging) borrowers is consistent with this implication of our theory.

Our theoretical framework is also well-suited to study effects of recent developments in the consumer-credit marketplace on emerging and established borrowers. For example, it may be used to examine the implications of the use of non-traditional sources of data for evaluating credit applications (these may range from social-media to phone-company data).[54] In our framework, these innovations can be modeled either as changes in the conditional correlation of signals across lenders or alternatively as changes in the precision of lenders' signals. Our model allows us to explicitly consider these possibilities (keeping in mind that predictions may depend on the exact information structure one assumes) and infer how these changes may be impacting consumer credit, particularly for emerging borrowers. We leave this extension for future work.

[54] Berg et al. (2020) point out that the digital footprint of an online shopper can be more informative of their default probability than their credit score.

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Appendices

A Additional Data Analysis

A.1 Card-Matching Algorithm

In the data we have obtained, for each individual and each date, we observe card-level data (balances and credit limits) for up to five credit cards. These cards are ordered (e.g. card 1, card 2, etc.) by the size of the balance and so card 1 in 2014 may not correspond to card 1 in 2015 and so forth. To link cards over time, we use an account status indicator provided to us by TransUnion that reflects each existing card's status over each of the past 24 months. In each month, a card may have a transactor ("T"), revolver ("R"), or an inactive ("I") status.[55] These monthly indicators yield a string of 1 to 24 characters for each card-year observation in our database. We match cards across time by seeking matches in a card's month 13-24 history in one year to the month 1-12 histories for the same borrower's cards in the previous year. We construct card linkages using the following criteria: (i) the two sequences exactly match and (ii) none of the borrower's other cards in the previous year are an exact match. The key qualitative results do not depend on the exact details of this matching algorithm, with looser matching criteria (allowing matches if 90% of the month observations agree) producing very similar results.

To test the match quality of our linkage algorithm, Table A1 displays coefficients from several regressions of card matching. In all columns, the left-hand-side variable is an indicator that a given card with at least a 12-month history was matched to a card in the previous period. In the first column, we see the overall match rate is 81%. The second column demonstrates that cards owned by the emerging borrowers are 14pp more likely to be matched, leading to an 90% match rate for emerging borrowers. The discrepancy in match rates for emerging and established borrowers is not too surprising as emerging borrowers have many fewer cards on average, and so the chance that a card rotates out of the five observable cards or has the same history as another card in the borrower's portfolio are lower than for cards owned by an emerging borrower. Column 3 demonstrates that successful card linkage is only weakly correlated with borrowers' observable characteristics (measured at the later of the two time points). We see that cards with the largest balance are 6.2pp more likely to have been matched and the likelihood that a card is not matched increases by 2.3pp for each open card, both of which are consistent with the fact

[55] For a small number of observations, we observe missing months, which we classify as a 4th type for the purpose of card matching.

that cards may rotate out of the set of five observable cards if five of the borrower's other cards carry a higher balance. Overall, the magnitudes of the coefficients in column 3 suggest that the imperfect matching algorithm is unlikely to generate a significantly selected subsample of the data, with the caveat that match rates differ significantly across samples (emerging and established).

Table A1: Balance of Card Matching

	(1) Card matched (0/1)	(2) Card matched (0/1)	(3) Card matched (0/1)
Emerging sample		0.138	0.019
		(0.000)	(0.001)
Years since first trade			-0.0025
			(0.0000)
Total credit line (1,000s)			-0.0000
			(0.0000)
Monthly mortgage payment (1,000s)			-0.0001
			(0.0002)
Any mortgage			-0.0114
			(0.0006)
Any student loans			0.005
			(0.001)
Any auto trade			0.000
			(0.001)
Year 3			0.012
			(0.001)
Year 4			0.023
			(0.001)
Months since newest trade open			0.000
			(0.000)
Number open trades			-0.0008
			(0.0001)
Ever been delinquent			-0.0127
			(0.0006)
Months since last delinquent			0.000
			(0.000)
Number open cards			-0.0225
			(0.0002)
Number cards with a pos. balance			-0.0075
			(0.0002)
Ever bankrupt (public record)			0.025
			(0.001)
Mortgage value (100,000s)			0.004
			(0.000)
Card with largest balance			0.062
			(0.000)
Constant	0.813	0.771	0.917
	(0.000)	(0.000)	(0.001)
N	3,191,780	3,191,780	3,191,780
R^2	0.000	0.027	0.080

Notes: A unit of observation is a card. The dependent variable is dummy for a card being matched to a previous card, conditional on the card history of 12 months or longer. Robust standard errors are reported in parentheses.

A.2 Incumbent vs. New Card Credit Limit Increases: Regressions

The results presented in subsection 2.3 in the main text were based on the total credit limit aggregated across cards. Using our card-level data and the matching algorithm described in the previous subsection, we further investigate the interaction between credit growth on existing cards and opening a new card at the individual card level. In Table A2, we examine the determinants of credit-limit growth of borrowers' existing or incumbent cards. Specifically, we regress annual growth in card-level credit limits onto the indicated variables. We show that credit-limit growth of incumbent cards is stronger for emerging borrowers than for established borrowers. We also find that for both emerging and established borrowers, opening a new card raises the growth rate of credit on incumbent cards. Finally, this latter effect is strongest for emerging borrowers as we find a statistically significant coefficient for the interaction term between emerging borrowers and borrowers who open a new card. We show that these relationships are robust when controlling for other aspects of the borrower's credit report.

Table A2: Impact of New Card on Percent Increase of Incumbent Card Credit Limit

	(1)	(2)	(3)	(4)	(5)	(6)
Emerging (0/1)	0.355***	0.646***	0.636***	0.479***	0.733***	0.019
	(0.006)	(0.018)	(0.018)	(0.022)	(0.021)	(0.041)
Opened new card (0/1)	0.216***	0.121***	0.113***	0.113***	0.394***	0.119**
	(0.006)	(0.008)	(0.009)	(0.008)	(0.025)	(0.049)
Emerging x New card	0.899***	0.649***	0.636***	0.628***	0.376***	0.653***
	(0.019)	(0.027)	(0.027)	(0.027)	(0.035)	(0.055)
No. bank inquiries past 12 months			0.029***	0.010**		
			(0.005)	(0.005)		
Utilization (pp)			0.000	-0.001***		
			(0.000)	(0.000)		
Total credit line (1,000s)				0.000***		
				(0.000)		
Any mortgage				-0.057***		
				(0.010)		
Any auto trade				-0.024**		
				(0.010)		
Any student loans				0.028***		
				(0.010)		
Vantage score (100s)				-0.184***		
				(0.011)		
Card with largest balance (0/1)				-0.204***		
				(0.010)		
Card balance (1,000s)				-0.060***		
				(0.003)		
Constant	0.300***	0.407***	0.392***	2.039***	0.321***	1.034***
	(0.002)	(0.006)	(0.007)	(0.088)	(0.013)	(0.038)
Sample						
Full	X					
Opened card +/- 8 months		X	X	X	X	X
Only 1 card					X	
Card less than 18 mo.						X
N	539,006	189,208	189,208	189,208	51,269	46,378
R^2	0.037	0.051	0.051	0.067	0.058	0.023

Notes: Each column displays coefficents from a regression of card level credit limit growth onto row variables. Growth is measured as percent growth (where a value of 1 = 100% growth) between 2014 and 2015. Controls are all measured in 2014, except inquiries, which are measured between 2014 and 2015. The new card variable is defined as a dummy that a new card was opened after September 2014 and on or before September 2015. Clustered standard errors are in parentheses.

As we have already mentioned before, we must be cautious to not necessarily interpret the described results as causal. For example, borrowers who have opened a new card likely have a higher demand for credit, which could also explain the expansion of the incumbent credit. To narrow the potential mechanisms which may explain these correlations, columns (2)–(6) in Table A2 consider the determinants of credit growth only for borrowers who open a new card within 8 months (before or after) of our observation period in September 2015. We expect that borrowers who have recently opened a new card are selected similarly to borrowers who have not yet, but soon will open a new card. Comparing columns (1) and (2), we find that the correlation between new cards and incumbent card growth remains significant and positive, particularly for emerging borrowers. When incorporating additional controls that are also likely to be correlated with higher demand for credit—e.g. in column (3) we include the number of recent bank inquiries as well as card utilization—we find that this result is robust.

A.3 Additional Tables and Figures

Table A3: Baseline Credit Lines and Balances: Medians

	Emerging	Emerging with credit card	Established	Established with credit card
Credit Line				
All (no mortgage)	1,500	1,000	31,701	36,749
	[495k]	[263k]	[358k]	[307k]
Auto	12,949	13,898	21,662	22,480
	[67k]	[8k]	[123k]	[105k]
Credit card	800	800	18,220	18,220
	[263k]	[263k]	[301k]	[301k]
Mortgage	132,000	-	160,695	167,879
	[2k]		[126k]	[114k]
Retail	500	400	4,501	5,000
	[74k]	[11k]	[217k]	[194k]
Student	2,750	2,750	20,460	22,500
	[66k]	[5k]	[59k]	[45k]
Balance				
All (no mortgage)	967	376	8,939	8,809
	[399k]	[191k]	[329k]	[286k]
Auto	12,572	13,402	14,125	14,444
	[67k]	[8k]	[123k]	[105k]
Credit card	319	319	2,369	2,369
	[186k]	[186k]	[268k]	[268k]
Mortgage	130,127	-	139,999	146,124
	[2k]		[126k]	[114k]
Retail	277	185	551	551
	[48k]	[7k]	[125k]	[112k]
Student	2,751	2,751	17,158	18,338
	[66k]	[5k]	[59k]	[45k]

Notes: The table reports the median amount of credit or balance in USD, measuring at the baseline observation (2014). Numbers of observations are in brackets. Cells representing less than 0.1% of the sample (less than 500 observations) are excluded. Means are conditional on having the credit type. Credit limits and balances are taken from trades verified in the preceding 12 months.

Table A4: Growth Rate of Aggregate Credit Limit from 2014 to 2015, %

	Sample 1		Sample 2		Sample 3		Sample 4		Sample 5	
	Growth	No. obs	Growth	No. obs	Growth	No. obs	Growth	No. obs	Growth	No. obs
Emerging										
Total	66.61	227,789	58.69	217,240	36.80	197,954	30.88	163,631	63.60	138,877
Cond. new card	253.17	57,837	226.38	55,860	250.11	29,573	134.57	25,182	230.27	35,728
Cond. no new card	24.13	169,952	22.58	161,380	19.88	168,381	12.31	138,449	26.07	103,149
Incumbent cards, cond. new card	165.46	57,837	142.60	55,860	82.64	29,573	85.11	25,182	150.16	35,728
Established										
Total	3.96	293,602	2.83	256,197	0.90	213,537	1.31	237,498	2.98	198,869
Cond. new card	24.00	61,598	31.32	45,437	30.72	25,514	26.12	35,587	30.55	34,616
Cond. no new card	−2.20	232,004	−2.55	210,760	−2.69	188,023	−3.23	201,911	−2.36	164,253
Incumbent cards, cond. new card	4.73	61,598	5.75	45,437	−5.49	25,514	2.52	35,587	5.09	34,616

Notes: Sample 1 includes all borrowers with non-missing credit limits in 2014 and 2015 or borrowers with non-missing credit limits in 2014 who have zero cards in 2015. Sample 2 adds the restriction that borrowers with credit limits in 2014 and 2015 have no more than five cards in 2015. Sample 3 includes only those borrowers for whom we can measure the 2015 credit limit on every new card opened between 2014 and 2015. Sample 4 includes borrowers from Sample 2 whose 2015 balances are smaller than their 2014 credit limits. Sample 5 includes borrowers from Sample 2 whose 2015 utilization is smaller than 50%.

Table A5: Aggregate Credit Limit Evidence

	Growth rate	2014 Average	2015 Average	No. obs
Emerging				
All	58.69	2,812	4,463	217,240
Cond. new card	226.38	1,938	6,325	55,860
Cond. no new card	22.58	3,115	3,819	161,380
Incumbent cards, cond. new card	142.60	1,938	4,701	55,860
Established				
All	2.83	22,641	23,282	256,197
Cond. new card	31.32	20,302	26,661	45,437
Cond. no new card	−2.55	23,145	22,554	210,760
Incumbent cards, cond. new card	5.75	20,302	21,470	45,437

Notes: Growth observations are from September 2014–September 2015.

Table A6: Growth Rate of Aggregate Available Credit Limit, %

	Sample 1		Sample 2		Sample 3		Sample 4		Sample 5	
	Growth	No. obs	Growth	No. obs	Growth	No. obs	Growth	No. obs	Growth	No. obs
Emerging										
Total	57.82	227,789	49.31	217,240	30.68	189,151	34.10	154,828	69.39	138,877
Cond. new card	251.84	57,837	221.62	55,860	232.06	29,198	160.14	24,807	251.78	35,728
Cond. no new card	15.58	169,952	13.99	161,380	15.28	159,953	11.51	130,021	30.28	103,149
Incumbent cards, cond. new card	168.55	57,837	142.38	55,860	72.87	29,198	104.39	24,807	166.49	35,728
Established										
Total	3.69	293,602	2.47	256,197	0.82	209,966	2.10	233,927	4.54	198,869
Cond. new card	24.37	61,598	32.54	45,437	31.06	25,403	30.35	35,476	34.35	34,616
Cond. no new card	−2.76	232,004	−3.16	210,760	−2.86	184,563	−3.12	198,451	−1.09	164,253
Incumbent cards, cond. new card	5.07	61,598	6.27	45,437	−5.67	25,403	5.12	35,476	7.81	34,616

Notes: Sample 1 includes all borrowers with non-missing credit limits in 2014 and 2015 or borrowers with non-missing credit limits in 2014 who have zero cards in 2015. Sample 2 adds the restriction that borrowers with credit limits in 2014 and 2015 have no more than five cards in 2015. Sample 3 includes only those borrowers for whom we can measure the 2015 credit limit on every new card opened between 2014 and 2015. Sample 4 includes borrowers from Sample 2 whose 2015 balances are smaller than their 2014 credit limits. Sample 5 includes borrowers from Sample 2 whose 2015 utilization is smaller than 50%.

Table A7: Aggregate Available Credit Evidence

	Growth Rate	2014 Average	2015 Average	No. obs
Emerging				
All	49.31	2,177	3,251	217,240
Cond. new card	221.62	1,441	4,633	55,860
Cond. no new card	13.99	2,432	2,773	161,380
Incumbent cards, cond. new card	142.38	1,441	3,492	55,860
Established				
All	2.47	18,400	18,855	256,197
Cond. new card	32.54	16,361	21,685	45,437
Cond. no new card	−3.16	18,840	18,245	210,760
Incumbent cards, cond. new card	6.27	16,361	17,388	45,437

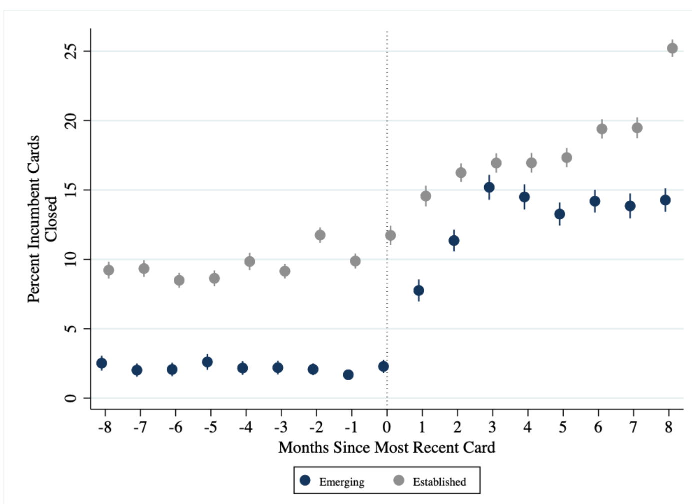
Notes: Growth observations are from September 2014–September 2015.

Table A8: Average Utilization, %

	No. obs	2014	2015	2016	2017
Emerging					
Total	176,467	30.03	36.58	36.28	36.49
Cond. new card	48,289	35.76	36.30	37.62	38.59
Cond. no new card	128,178	27.87	36.68	35.77	35.70
Established					
Total	236,528	26.68	26.53	25.66	25.39
Cond. new card	43,131	30.30	27.87	28.25	28.44
Cond. no new card	193,397	25.87	26.23	25.08	24.70

Notes: The sample includes all borrowers with non-missing credit limits in 2014 and 2015 and with no more than five cards in 2015, or borrowers with non-missing credit limits in 2014 who have zero cards in 2015. The sample also requires borrowers have observable utilization records in 2014, 2015, 2016 and 2017. "Cond. new card" ("Cond. no new card") selects borrowers who opened (did not open) a new card between October 2014 and September 2015.

Figure A1: Incumbent Credit Limit Event Study: Linear Probability Model, Closed Incumbent Cards



Notes: The figure displays the share of incumbent cards (cards which existed in 2014) which were closed by 2015 by months since an individual opened their most recent card. The sample is restricted to individuals who opened a card +/- 8 months of September 2015. Timing of the credit card opening is determined by the earliest month implied by the account status indicator and constructed card linkages.

Table A9: Incumbent Credit Limit Event Study

Months since most recent card	Emerging	Established
-8	82.29	36.48
-7	80.84	34.24
-6	91.89	37.11
-5	93.4	36.07
-4	96.14	40.2
-3	89.34	38.97
-2	101.05	40.67
-1	100.35	38.54
0	113.65	42.4
1	198.96	59.18
2	225.24	62.42
3	226.26	64.08
4	214.42	60.52
5	209.7	63.68
6	166.46	51.22
7	175.02	54.62
8	164.87	46.71

Notes: The table shows the average credit growth from incumbent cards (those cards with an account status indicator of 12 characters or longer) and new cards (account status indicator is shorter than 12 characters). The sample is restricted to emerging borrowers who have an open credit card in each of the four periods. The average credit from new cards is calculated by totaling the card-level credit limits within individual and then averaging across individuals. The increase in credit from incumbent cards is computed as the increase in total credit card limit minus the total credit on new cards. Missing card-level credit limits are set to the aggregate credit limit if an individual only had one card. If credit limits on new cards are still missing, but card-level credit limits on old cards are known for all cards and the individual has five or fewer cards, the total credit on new cards is calculated as total credit card limit minus the credit limit of incumbent cards. If missing credit limits on new cards cannot be imputed by one of these two methods, the observation is dropped from the period in question.

Table A10: Size of New Card and Future Delinquency: Probit

	Emerging				Established			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
New card limit '14-'15 (share '14 lim)	-0.0010***	-0.0011***	-0.0009***	-0.0019***	0.0038***	-0.0004	-0.0005	-0.0003
	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0003)	(0.0003)	(0.0004)	(0.0004)
Opened new card '14-'15 (0/1)	0.0438***	0.0354***	0.0524***	0.0531***	0.0117***	0.0095***	0.0179***	0.0194***
	(0.0013)	(0.0012)	(0.0013)	(0.0017)	(0.0011)	(0.0010)	(0.0011)	(0.0012)
Vantage score '14		-0.0006***				-0.0008***		
		(0.0000)				(0.0000)		
Total credit card limit '14 ($1,000s)			-0.0068***	-0.0108***			-0.0018***	-0.0028***
			(0.0003)	(0.0015)			(0.0000)	(0.0000)
Total credit limit '14 ($1,000s, exc. mortgage)			0.0000	-0.0015			0.0000***	-0.0001***
			(0.0003)	(0.0015)			(0.0000)	(0.0000)
Incumb. card limit increase '14-'15 (share '14 lim)			-0.0060***	-0.0075***			-0.0008***	-0.0008***
			(0.0001)	(0.0001)			(0.0002)	(0.0002)
Student loan '14 (0/1)			-0.0092***	-0.0069			0.0163***	0.0094***
			(0.0031)	(0.0043)			(0.0013)	(0.0013)
Mortgage '14 (0/1)			-0.0216	-0.0294			-0.0014	-0.0093***
			(0.0160)	(0.0196)			(0.0009)	(0.0010)
Auto loan '14 (0/1)			0.0173***	0.0146**			0.0175***	0.0123***
			(0.0055)	(0.0066)			(0.0010)	(0.0010)
Total credit balance '14 ($1,000s, exc. mortgage)				0.0017				0.0001***
				(0.0016)				(0.0000)
Total credit card balance '14 ($1,000s)				0.0097***				0.0039***
				(0.0016)				(0.0001)
N	206,951	206,951	206,951	142,966	285,529	285,529	285,529	257,123
Sample avg. delinquency rate	0.052	0.052	0.052	0.062	0.060	0.060	0.060	0.062
Pseudo R^2	0.0164	0.1104	0.0741	0.0866	0.0026	0.2280	0.0624	0.1143

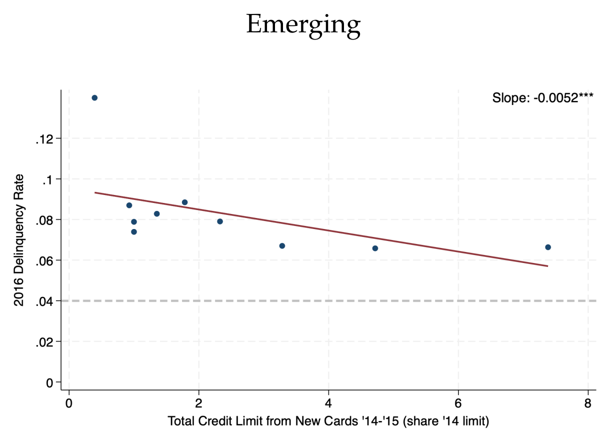
Notes: The table displays marginal effects from a probit regression of a dummy for any card trade more than 90 days past due in 2016 onto the indicated row variables. The samples are conditional on having an open bank card in 2014 and 2016. The new card limit corresponds to the total card-level credit limit on all cards opened between 2014 and 2015. Utilization is measured across all open cards. Credit limits and balances are taken from trades verified in the last 12 months. The reported average marginal effects of the new card limit reflect the average marginal effects of an increase in a borrower's new card limit computed only among the population of borrowers who opened a new card from 2014 to 2015.

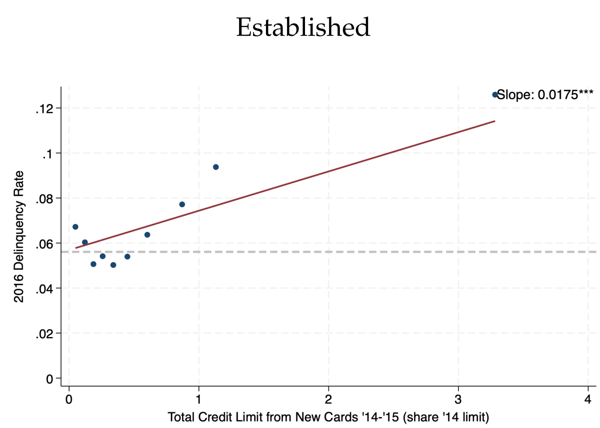
Table A11: New Card and Future Delinquency: Summary Statistics

	Emerging				Established			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Delinquency rate	0.0520	0.0520	0.0520	0.0617	0.0597	0.0597	0.0597	0.0620
	(0.2221)	(0.2221)	(0.2221)	(0.2406)	(0.2369)	(0.2369)	(0.2369)	(0.2412)
New card limit ’14-’15 (share ’14 lim)	1.0282	1.0282	1.0282	1.1137	0.2000	0.2000	0.2000	0.1856
	(3.6481)	(3.6481)	(3.6481)	(3.5999)	(1.1310)	(1.1310)	(1.1310)	(1.0088)
Opened new card ’14-’15 (0/1)	0.2968	0.2968	0.2968	0.3237	0.2781	0.2781	0.2781	0.2849
	(0.4568)	(0.4568)	(0.4568)	(0.4679)	(0.4481)	(0.4481)	(0.4481)	(0.4514)
Vantage score ’14		662.2355				749.5848		
		(58.4729)				(70.0632)		
Total credit card limit ’14 ($1,000s)			2.6280	2.9557			28.0801	29.8699
			(6.0789)	(6.8343)			(29.9094)	(30.6604)
Total credit limit ’14 ($1,000s, exc. mortgage)			3.2886	3.7010			51.7829	54.7067
			(7.1602)	(7.8280)			(56.2607)	(57.5859)
Incumb. card limit increase ’14-’15 (share ’14 lim)			3.4848	3.5630			0.3822	0.3628
			(8.3746)	(8.4258)			(2.1024)	(1.8777)
Student loan ’14 (0/1)			0.0225	0.0209			0.1479	0.1529
			(0.1482)	(0.1431)			(0.3550)	(0.3599)
Mortgage ’14 (0/1)			0.0007	0.0007			0.3817	0.3976
			(0.0255)	(0.0271)			(0.4858)	(0.4894)
Auto loan ’14 (0/1)			0.0310	0.0360			0.3472	0.3620
			(0.1734)	(0.1864)			(0.4761)	(0.4806)
Total credit balance ’14 ($1,000s, exc. mortgage)				1.5463				20.2769
				(4.7058)				(37.3197)
Total credit card balance ’14 ($1,000s)				0.8604				5.9474
				(2.9816)				(9.7838)
N	206,951	206,951	206,951	142,966	285,529	285,529	285,529	257,123

Notes: The table displays mean values and standard deviations (in parentheses) for the variables used in the probit regressions in Tables 5 and A10. The samples are conditional on having an open bank card in 2014 and 2016. The new card limit corresponds to the total card-level credit limit on all cards opened between 2014 and 2015. Utilization is measured across all open cards. Credit limits and balances are taken from trades verified in the last 12 months.

Figure A2: Size of New Card and Future Delinquency: Graphical Representation





Notes: Dots represent equally sized bins among borrowers who opened a new card between 2014 and 2015. Gray lines represent the delinquency rates conditional on not opening a new card.

B Model Extension: Credit Lines

In this appendix, we propose a variation of the model in which instead of loans, the lenders offer credit lines. This variation of the model preserves the mechanism of the model in the main text. In particular, the most creditworthy borrowers build credit history by opening a credit line—which they may or may not utilize—and may end up with excessively large credit limits.

Consider the following changes to the environment. First, the borrower's intertemporal discount factor β is now a random variable—uncorrelated with her quality or lenders' signals—which is realized at the end of period I, after all of the contracting is done. Second, the contracts are now credit lines, which the borrower may choose to utilize (or not) at the end of period I upon realization of the discount factor. The discount factor takes one of two values: $\beta = 0$ with probability π and $\beta = B \geqslant \bar{q}$ with probability $1 - \pi$.

The resulting equilibrium of this modified game is nearly identical to the equilibrium described in the paper. The prices are exactly the same as before, with loan sizes being replaced by credit limits. The equilibrium strategies (lenders' offers and borrowers' acceptance decisions) remain unchanged. The only difference is that only borrowers with realized $\beta = 0$ utilize their credit lines, while individuals with $\beta = B$ do not borrow.[56]

Importantly, in the proposed extension, which allows for distinction between credit lines and credit balances, the key mechanism remains entirely the same as in the model in the main text. In particular, AA-borrowers open credit lines to facilitate information aggregation across lenders, and may overborrow. The prediction of "more dilution, lower default risk" is also the same as before.

C Comparative Statics and Welfare Implications

In this appendix, we use a numerical example to illustrate key comparative statics and welfare implications of the model described in Section 4. The parameter values we use are $e_\ell = 3.5$, $e_m = 9$, $e_h = 12$, $\alpha = 0.28$, $\delta = 0.82$, and we vary the signal precision ρ. The three equilibria that we described in subsection 4.2 arise for some values of ρ, while other equilibria arise for other values of ρ.[57]

[56]In this simple extension individual borrowers have either zero or 100% utilization of their credit limits, while the average (across borrowers) utilization is interior. In order to generate interior utilization for individual borrowers, we need to allow a realization of $\beta \in (0, \bar{q})$ *and* assume that the borrower's utility function is strictly concave so that the borrower chooses an interior level of utilization following such a realization of β.

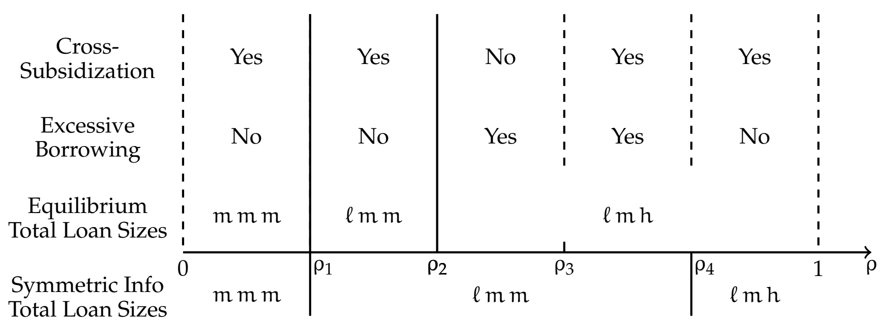
[57]While we do not provide full characterization of the off-path beliefs and strategies in all those equilibria, their construction is similar to the equilibria described here, and is available upon request.

In subsection C.1, we restrict our attention to equilibria with credit-history building and show how equilibrium outcomes and credit-history building costs and benefits vary with the signal precision ρ. In subsection C.2 we analyze whether equilibrium with or without credit-history building survives our selection criterion, how this selection varies with ρ, and discuss welfare implications. In particular, we illustrate that availability of credit records is not necessarily ex-ante welfare improving and that welfare may be non-monotone in the signal precision.

C.1 Comparative Statics of Equilibrium Outcomes under Credit-History Building

Figure A3 illustrates (from the bottom to the top) equilibrium outcomes under symmetric information, equilibrium outcomes in our game under credit-history building, and the presence of excessive borrowing and cross-subsidization. For equilibrium outcomes, we only report the total loan sizes, using the following notation: xyz, with $x, y, z \in \{\ell, m, h\}$, meaning that BB-borrower's total loan is φe_x, AB's is φe_y, and AA's is φe_z. It is important to note that while equilibrium total loan sizes (measured as the loan face values) might be the same under symmetric and asymmetric information, the corresponding loan prices are typically not the same.

Figure A3: Credit-history-building equilibrium outcomes as functions of the signal precision ρ.



Notes: $\ell m h$ means φe_ℓ to BB-borrowers, φe_m to AB-borrowers, φe_h to AA-borrowers.

Given our parametric assumptions, the equilibrium outcome with uninformative signals is mmm, i.e., a medium loan for all borrowers. Moreover, we assume that with arbitrarily informative signals (ρ close to one), under symmetric information there is full separation by loan size, i.e., we get the ℓmh outcome. The thresholds $\rho_1, \ldots, \rho_4$ displayed on the figure mark switches in the outcomes and incidence of excessive borrowing and cross-subsidization, and will be convenient later to match to the corresponding thresholds in Figure A4.[58]

Consider how equilibrium outcomes change as ρ falls from 1 to 0. For ρ high enough, the equilibrium depicted in the figure (columns 4 and 5) is the ℓmh equilibrium with cross-subsidization described in subsection 4.2.2. When ρ is sufficiently close to one, there is no excessive borrowing in this case as AA-borrowers take on a large loan under symmetric information (column 5). Hence, there is no cost of credit-history building when ρ is sufficiently large.

As ρ decreases just below ρ_4, the size of the loan that an AA-borrower takes in the symmetric-information equilibrium falls from a large loan to a medium loan (column 4). The reason is that as ρ declines, an AA-borrower's perceived probability of receiving a high endowment in period II declines. That is, AA-borrowers become more pessimistic about their endowment process and choose to borrow less in period I (see the bottom row of Figure A3).

Note that the switch from a large to medium loan by an AA-borrower does not happen at the same value of ρ in the asymmetric-information environment (the symmetric-information outcome switches at ρ_4 while the asymmetric-information outcome switches at ρ_2). But in the equilibrium with credit-history building, the choice in the second stage is over the top-up portion of the loan, and AA-borrowers end up with a large instead of a medium loan. This outcome reflects the excessive-borrowing feature that we have discussed earlier.

As ρ decreases further, the likelihood that an AA-borrower repays a large loan falls and with it the price, q_h^{AA}. On the other hand, q_m^{AB} remains unchanged. The leads to a violation of the cross-subsidization condition (10) for sufficiently low signal precision. That is, for low enough ρ, AB-borrowers would no longer receive a subsidy if they were to take a stage-1 loan and thus prefer to wait for an actuarially fairly priced loan in stage 2. Hence at ρ_3 we switch to an equilibrium without cross-subsidization, where only AA-borrowers accept an early loan—column 3 in the figure.

A further decrease in ρ makes AA-borrowers' endowment prospects less and less favorable, which causes their price of a large loan to fall. Ultimately, at ρ_2 these borrowers prefer to switch from a large to a medium-size loan (column 2) in equilibrium. Of course, as that happens, AB-borrowers start accepting the early loan, and we again have cross-subsidization.

Finally, as ρ gets sufficiently close to zero, the information content of the signals vanishes. As a consequence, borrowers with different signal combinations have sufficiently similar endowment prospects. In equilibrium (as well as in the symmetric-information

[58]On Figure A3, the x-axis is not done to scale.

benchmark), all borrowers obtain a medium loan. (This happens for signal precisions below ρ_1.) Note that cross-subsidization only happens between AA- and AB-borrowers.

As Figure A3 illustrates, cross-subsidization takes place for large enough and small enough values of the signal precision, while excessive borrowing occurs for intermediate values of signal precision. Moreover, cross-subsidization and excessive borrowing can occur simultaneously or one at a time.

In the next subsection we explore whether AA-borrowers prefer equilibria with or without credit-history building. As we discussed earlier, equilibria without credit-history building feature even more cross-subsidization—it applies to the entire loan, not just the first-stage portion of it. So limiting cross-subsidization is one of the benefits of credit-history building for AA-borrowers. The potential cost is excessive borrowing—equilibria without credit-history building do not feature it because all borrowing happens at once. We show that when the cost of excessive borrowing is particularly severe, the selected equilibrium does not feature credit-history building.

C.2 Welfare Implications

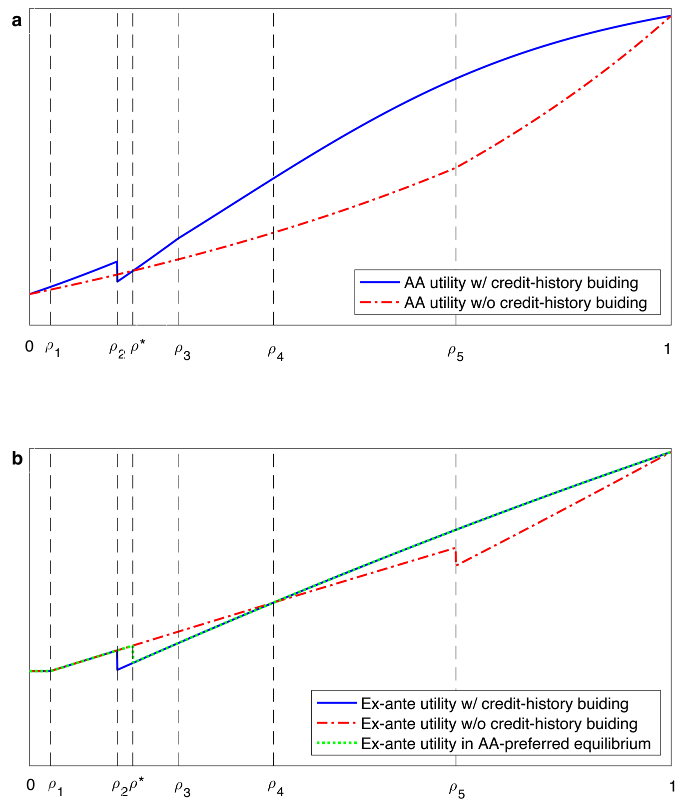
Using our numerical example, we now illustrate that (i) AA-borrowers may prefer an equilibrium without credit-history building to one with credit-history building; (ii) ex-ante welfare (before the signals are realized) may be higher without credit history building even when AA-borrowers prefer credit-history building (thus making credit-history building a selected equilibrium outcome); (iii) ex-ante welfare may be non-monotone in the precision of information.

These points are illustrated on Figure A4, which plots utilities in the equilibria with and without credit-history building. Panel **a** displays the utility of AA-borrowers in the equilibrium with credit-history building (the blue solid line, corresponding to utility in equilibrium from Figure A3) and in the equilibrium without credit-history building (red dash-dotted line). By construction, the AA's utility in the selected (preferred by AAs) equilibrium is the upper envelope of the two lines. Panel **b** displays the ex-ante (before the signals are realized) utility of borrowers in the equilibria with and without credit-history building (blue solid and red dash-dotted lines, respectively), and the ex-ante utility in the equilibrium preferred by AA-borrowers (green dotted line).

The thresholds $\rho_1, \ldots, \rho_4$ are the same as in Figure A3, while thresholds ρ^* and ρ_5 are new.[59] The allocation without credit-history building is mmm for $\rho \in (0, \rho_1]$, ℓmm on $[\rho_1, \rho_5]$, and ℓhh on $[\rho_5, 1)$. The threshold ρ_5 marks the point at which the equilibrium

[59]Unlike in Figure A3, the x-axis partition is now done to scale.

Figure A4: AA-borrowers' and ex-ante utilities as functions of the signal precision ρ.



allocation without credit-history building switches from a medium to a large loan for borrowers with an A signal. The threshold ρ^* marks a switch from no credit-history building to credit-history building.

The interesting thresholds here are ρ_2, ρ^*, and ρ_4. As shown on panel **a**, the AA's utility with credit-history building drops at ρ_2. To see why that happens, recall from Figure A3 the equilibrium outcome with credit-history building is ℓmm just below ρ_2 and ℓmh with excessive borrowing just above ρ_2. The reason for the drop is excessive borrowing. As a result, AA-borrowers prefer not building credit history just to the right of ρ_2.

The threshold ρ^* marks the value of precision at which AA-borrowers again prefer to build credit history.[60] Notice that the ex-ante utility in the selected equilibrium on panel **b**

[60]Between the equilibria with and without credit-history building, our equilibrium selection picks the one preferred by AA-borrowers. Notice that when the two equilibria yield exactly the same payoffs to AA-borrowers (which happens at ρ^*), both of them survive our selection. In this measure-zero case, these two equilibria are Pareto ranked: no credit-history building Pareto superior. AA-borrowers are indifferent between the two equilibria by assumption (despite receiving different loans), while AB-borrowers whose A signal comes from fuchsia lenders, and BB-borrowers end up with exactly the same loans in the two equilibria. On the other hand, AB-borrowers whose A signal comes from green lenders strictly prefer the equilibrium without credit-history building—they receive a more generous cross-subsidy on a larger loan in that equilibrium, resulting in the same utility as that of the AA-borrowers.

declines discretely at ρ^*. The reason is again excessive borrowing. Why does the expected utility drop even though AA's utility is continuous at ρ^*? This is because just to the right of ρ^* there is less cross-subsidization, which benefits AAs and compensates them for the reduction in utility due to excessive borrowing. (Cross-subsidization has no effect on the expected utility because it is a transfer from AAs to ABs.) At ρ_4, excessive borrowing stops. Therefore to the right of ρ_4 credit-history building becomes ex-ante efficient.

The above observations bring us to the following three points. First, both from the ex-ante perspective as well as from the point of view of AA-borrowers, credit-history building is not always desirable. The underlying reason is excessive borrowing, which comes from the borrower's inability to commit not to over-dilute the first-stage lender. Second, our equilibrium selection picks credit-history building too often—on the interval $[\rho^*, \rho_4]$ credit-history building is not desirable from the ex-ante perspective, but preferred by AA-borrowers, despite the excessive borrowing. The reason is that equilibrium without credit-history building features more cross-subsidization. The AA-borrowers dislike cross-subsidization, while the planner does not care about it.[61] Third, social welfare may be non-monotone in the signal precision. That is, more precise information—e.g., arising from an improvement in lenders' statistical models—does not necessarily make borrowers better off. The reason the welfare can drop is, once again, excessive borrowing.

To sum up, our simple model yields surprisingly complex predictions regarding the desirability of public credit records. For some parameter values (levels of signal precision $\rho > \rho_4$) having public credit records is strictly beneficial, while for other values ($\rho \in (\rho^*, \rho_4)$) it lowers ex-ante welfare. The possible welfare loss is due to debt dilution. Interestingly, in our environment public credit records do not prevent but instead encourage borrowing from multiple lenders that leads to debt dilution.

D Proofs and Equilibrium Constructions for the Simple Model

In this appendix, we provide a complete description of the equilibrium with credit-history building described in subsection 3.2 and prove the results stated there.

[61] With risk-neutral borrowers, cross-subsidization does not matter for the ex-ante utility. If the borrowers were risk averse, cross-subsidization would create insurance and increase the ex-ante utility. This would further strengthen the result that AAs build credit history too often from the ex-ante perspective.

D.1 Case with $\beta = 0$

Strategies

Period II: Borrowers repay their loan if and only if $x \leqslant \varphi e$.

Stage 2:
— The borrower accepts the largest loan offered (i.e., the one with the largest qx).
— Lenders who do not see any loans: offer φe_ℓ at $\bar{q}$.
— Lenders who see stage-1 loan (x, q): if the loan is from a different class of lenders and your signal is A, offer $(\varphi e_h - x)$ at q_h^{AA}; otherwise (B signal or loan from your own class) offer $(\varphi e_\ell - x)$ at $\bar{q}$ if $x < \varphi e_\ell$, offer nothing if $x \geqslant \varphi e_\ell$.

Stage 1: We restrict attention to offer sets that include $(\underline{x}, q_h^{AA})$ from A-lenders and some other offer (x, q).
— AA-borrowers: For loans (x, q), $x \leqslant \varphi e_h$, accept if $qx + q_h^{AA}\varphi(e_h - x) > q_h^{AA}\varphi e_h$, otherwise accept $(\underline{x}, q_h^{AA})$. For loans (x, q), $x > \varphi e_h$, accept if $qx > q_h^{AA}\varphi e_h$, otherwise accept if $(\underline{x}, q_h^{AA})$.
— AB-borrowers: For loans (x, q), $x \in [\underline{x}, \varphi e_h)$, offered by a B-lender,[62] accept the loan if $qx + q_h^{AA}(\varphi e_h - x) \geqslant \bar{q}\varphi e_\ell$. For loans (x, q), $x \in [\underline{x}, \varphi e_\ell)$, offered by an A-lender, accept the loan if $qx + \bar{q}(\varphi e_\ell - x) \geqslant \bar{q}e_\ell$ (i.e., if $q \geqslant \bar{q}$). For loans (x, q), $x \in [\varphi e_\ell, \varphi e_h]$, offered by an A-lender, accept the loan if $qx \geqslant \bar{q}\varphi e_\ell$. For loans (x, q), $x > \varphi e_h$, offered by any lender: accept the loan if $qx \geqslant \bar{q}\varphi e_\ell$.
— BB-borrowers: For loans (x, q), $x \geqslant \varphi e_\ell$, accept the loan if $qx \geqslant \bar{q}\varphi e_\ell$. For loans (x, q), $x < \varphi e_\ell$, accept the loan if $qx + \bar{q}(\varphi e_\ell - x) \geqslant \bar{q}e_\ell$ (i.e., if $q \geqslant \bar{q}$).
— A-lenders offer $\underline{x}$ at q_h^{AA}.
— B-lenders offer nothing.

Beliefs

Stage-2 off-path beliefs are as follows. For a B-lender who sees a stage-1 loan, their beliefs about the other class's signal do not matter. An A-lender who sees a stage-1 loan from the other class (of any size and any price) believes that the loan came from an A-lender.

Proof of Proposition 1. To establish that the construction above constitutes an equilibrium, we need to verify the players' incentives at every node. Period-II incentives are satisfied by construction. In stage 2 of period I, the incentives for the borrower are trivial given that $\beta = 0$. Furthermore, for the lenders, given the beliefs, there are no profitable deviations that would be accepted.

[62] The borrower deduces the signal of a deviating lender from the total number of offers she receives.

Consider stage-1 incentives for the borrower. For AA-borrowers, accepting the offer is better than rejecting the loan and getting a safe loan in stage 2 if and only if $q_h^{AA}e_h \geqslant \bar{q}e_\ell$, which is condition (1) in the text. AB-borrowers must prefer to not accept the loan offered by A-lenders. That follows directly from $\bar{q} > q_h^{AA}$. BB-borrowers have no offers. Off-equilibrium-path strategies for all borrowers are optimal by construction.

Consider stage-1 incentives for A-lenders. (i) Suppose an A-lender offers a loan with $q < q_h^{AA}$. It will be rejected by AA-borrowers, since they can get a better price. AB-borrowers cannot mislead anyone and thus accept such a loan only if $qx > \bar{q}\varphi e_\ell$. But that implies $x > \varphi e_\ell$, and thus the loan is never repaid. (ii) Suppose an A-lender offers a loan with $q \in (q_h^{AA}, \bar{q})$. It is always accepted by AAs. AB-borrowers only accept if $qx > \bar{q}\varphi e_\ell$, which requires $x > \varphi e_\ell$. Thus, such a loan definitely loses money, as it loses both on AAs and on ABs if they accept it. (iii) Suppose an A-lender offers a loan with $q \geqslant \bar{q}$. It would be accepted by everyone and lose money, since AAs sometimes do not pay them back. This shows that A-lenders do not want to deviate.

Consider stage-1 incentives for B-lenders. (i) Suppose a B-lender offers a loans with $q < \bar{q}$. If $x > \varphi e_\ell$, the loan is never repaid and must lose money. If $x \leqslant \varphi e_\ell$, the loan does not attract BBs. If the loan is accepted by ABs, it gets topped up to φe_h, and thus never gets repaid. (ii) Suppose a B-lender offers a loan with $q = \bar{q}$. It is is always accepted by ABs and topped up to a large loan, thus necessarily loses money. Thus B-lenders also do not want to deviate.

Since the only condition on parameter values in the above equilibrium construction is condition (1), the statement of the proposition follows. □

D.2 Case with $\beta > 0$

We modify the equilibrium construction relative to the $\beta = 0$ case in the previous subsection only for the borrower's strategies. The changes are as follows.

Stage 2:

– A borrower with a stage-1 loan x_1 accepts an offer (x_2, q) if and only if $qx_2 \geqslant \beta(E\max\{(1-\varphi)y, y - x_1\} - E\max\{(1-\varphi)y, y-(x_1+x_2)\})$. Among the available offers, she accepts one that maximizes $qx_2 + \beta(E\max\{(1-\varphi)y, y-(x_1+x_2)\} - E\max\{(1-\varphi)y, y-x_1\})$.

– A borrower without a stage-1 loan accepts an offer (x, q) if and only if $qx + \beta E\max\{(1-\varphi)y, y-x\} \geqslant \beta Ey$. Among the available offers, she accepts accepts one that maximizes $qx + \beta E\max\{(1-\varphi)y, y-x\}$ if and only if $qx + \beta E\max\{(1-\varphi)y, y-x\} \geqslant \beta Ey$.

Stage 1:

– AB-borrowers: For loans (x, q), $x \in [\underline{x}, \varphi e_h]$, offered by a B-lender, accept the loan if

$qx + q_h^{AA}(\varphi e_h - x) \geqslant \bar{q}\varphi e_\ell$. For loans (x, q), $x \in [\underline{x}, \varphi e_\ell)$, offered by an A-lender, accept the loan if $qx + \bar{q}(\varphi e_\ell - x) \geqslant \bar{q}e_\ell$ (i.e., if $q \geqslant \bar{q}$). For loans (x, q), $x \in [\varphi e_\ell, \varphi e_h]$ offered by an A-lender, accept the loan if $qx \geqslant \bar{q}\varphi e_\ell$. For loans (x, q), $x > \varphi e_h$, offered by any lender, accept the loan if $qx \geqslant \bar{q}\varphi e_\ell$.
– AA- and BB-borrowers' strategies are the same as before.

Proof of Proposition 2. To establish that the construction above constitutes an equilibrium, we need to verify the players' incentives at every node. Period-II incentives are satisfied by construction. Consider incentives in stage 2 of period I. For the lenders, given the beliefs, there are no profitable deviations that would be accepted. Consider the borrower's incentives. ABs and BBs prefer to accept the stage-2 loan $(\varphi e_\ell, \bar{q})$ as long as $\beta < \bar{q}$. AAs top up to large loan. The expected utility from accepting the top-up is $\varphi e_h q_h^{AA} + \beta\{\Pr(e_h|AA)(1-\varphi)e_h + [1 - \Pr(e_h|AA)](1-\varphi)e_\ell\}$. The expected utility from rejecting it is $\underline{x}q_h^{AA} + \beta\{\Pr(e_h|AA)(e_h - \underline{x}) + [1 - \Pr(e_h|AA)](e_\ell - \underline{x})\}$. The former exceeds the latter if and only if $q_h^{AA}(\varphi e_h - \underline{x}) \geqslant \beta(\Pr(e_h|AA)\varphi e_h + (1-\Pr(e_h|AA))\varphi e_\ell - \underline{x})$, which can be rewritten as $\bar{q}\Pr(e_h|AA)(\varphi e_h - \underline{x}) \geqslant \beta(\Pr(e_h|AA)\varphi(e_h - e_\ell) + \varphi e_\ell - \underline{x})$ or $\bar{q}\Pr(e_h|AA)(\varphi e_h - \underline{x}) - \beta(\varphi e_\ell - \underline{x}) \geqslant \beta \Pr(e_h|AA)\varphi(e_h - e_\ell)$. Further manipulation yields $\bar{q}\varphi\,(\Pr(e_h|AA)e_h - e_\ell) + \varphi\bar{q}e_\ell - \bar{q}\Pr(e_h|AA)\underline{x} - \beta(\varphi e_\ell - \underline{x}) \geqslant \beta\Pr(e_h|AA)\varphi(e_h - e_\ell)$, which is equivalent to $\bar{q}\varphi(\Pr(e_h|AA)e_h - e_\ell) + (\varphi e_\ell - \underline{x})(\bar{q} - \beta) + \bar{q}\underline{x}(1 - \Pr(e_h|AA)) \geqslant \beta\Pr(e_h|AA)\varphi(e_h - e_\ell)$. This is implied by condition (2), $\beta \leqslant \bar{q}$, and $\underline{x} \leqslant \varphi e_\ell$.

Consider stage-1 incentives. For the AA-borrower, expected utility from following the equilibrium strategy is $\varphi e_h q_h^{AA} + \beta\{\Pr(e_h|AA)(1-\varphi)e_h + [1 - \Pr(e_h|AA)](1-\varphi)e_\ell\}$. Her expected utility from deviating by taking no loan in stage 1 and taking a small risk-free loan in stage 2 is $\varphi e_\ell\bar{q} + \beta\{\Pr(e_h|AA)(e_h - \varphi e_\ell) + [1 - \Pr(e_h|AA)](1-\varphi)e_\ell\}$. The former exceeds the later if and only if condition (2) holds. AB-borrowers must prefer to not accept the loan offered by A-lenders. That follows directly from $\bar{q} > q_h^{AA}$. BB-borrowers have no offers. Off-equilibrium-path strategies for all borrowers are optimal by construction. Stage-1 incentives for A-lenders are analogous to those in the proof of Proposition 1.

Since the only conditions on parameter values in the above equilibrium construction are $\beta < \bar{q}$ and condition (2), the statement of the proposition follows. □

D.3 Private Information of the Borrower

In this subsection, we analyze an extension in which the borrower observes additional private information about her endowment distribution. Suppose she observes a hidden state $z \in \{z_n, z_p\}$ such that $\Pr(e = e_h|s = g, z = z_p) > \Pr(e = e_h|s = g, z = z_n)$ while $\Pr(e = e_\ell|s = b, z) = 1$ for any $z \in \{z_n, z_p\}$. That is, as before, $\Pr(g|AB, z) = \Pr(g|BB, z) = 0$

for any z but $\Pr(e_h|AA, z_p) > \Pr(e_h|AA, z_n) > 0$. Denote $\Pr(z = z_p) = \zeta$. As in the previous subsection, we assume that $\beta > 0$ (the analysis with $\beta = 0$ is trivial, as we explain in the main text.) We will refer to an AA-borrower with $z = z_i$, $i \in \{n, p\}$, as an AAi-borrower. For brevity, we also denote $\Pr(g|AA, z_i) = \Pr(g|AAi)$, $i \in \{n, p\}$.

The equilibrium construction we propose mimics that with $\beta > 0$ analyzed in the previous subsection. The key change is that lenders in stage 2 when facing an AA-borrower offer one of the least cost-separating loans. Stage-1 strategies, and strategies of B-lenders and B-borrowers are as before. A-lenders in stage 1 still offer $\underline{x}$ at $q_h^{AA} = \bar{q}\Pr(e_h|AA) = \bar{q}[\zeta \Pr(e_h|AAp) + (1-\zeta)\Pr(e_h|AAn)]$. In stage 2, A-lenders who see a stage-1 loan from the other class of lenders offer one of two loans:[63]

– a stage-2 loan for AAn is $x_2^n = \varphi e_h - \underline{x}$ at the price $q_h^{AAn} = \bar{q}\Pr(e_h|AAn)$.

– a stage-2 loan for AAp is x_2^p at the price $q_h^{AAp} = \bar{q}\Pr(e_h|AAp)$, where x_2^p solves $q_h^{AAn}x_2^n + \beta\Pr(e_h|AAn)e_h(1-\varphi) = q_h^{AAp}x_2^p + \beta\Pr(e_h|AAn)(e_h - x_2^p)$, which simplifies to

$$x_2^p = (\varphi e_h - \underline{x})\,\frac{\bar{q}\Pr(e_h|AAn) - \beta\Pr(e_h|AAn)}{\bar{q}\Pr(e_h|AAp) - \beta\Pr(e_h|AAn)}. \tag{A1}$$

This equation is intuitive. If $\beta = \bar{q}$ then surplus for the n-type is zero and hence no surplus can be given to the p-type (i.e. $x_2^p = 0$). We also see that $x_2^p < x_2^n$ and that this distortion is increasing in $\Pr(e_h|AAp) - \Pr(e_h|AAn)$ (since $\bar{q} \geqslant \beta$).

Stage-2 strategies are analogous to those in the previous section, but with appropriate modifications of expectations given AAn's and AAp's period-2 endowment probabilities.

Proceeding with incentives, the first condition is that p-type AA-borrowers prefer this loan to a safe loan:

$$q_h^{AA}\underline{x} + q_h^{AA,z_p}x_2^p + \beta\Pr(e_h|AAp)(e_h - \underline{x} - x_2^p) \geqslant \bar{q}\varphi e_\ell + \beta\Pr(e_h|AAp)(e_h - \varphi e_\ell). \tag{A2}$$

This condition is an analog of condition (2). We show in Lemma 1 below that if AAp-borrowers prefer credit-history building on these terms to a safe loan, then so do AAn-borrowers.

The second condition is that pooling AAp- and AAn-borrowers in stage 2 is not profitable (i.e., does not attract AAp-borrowers):

$$q_h^{AAp}x_2^p + \beta\Pr(e_h|AAp)(e_h - \underline{x} - x_2^p) \geqslant q_h^{AA}(\varphi e_h - \underline{x}) + \beta\Pr(e_h|AAp)(1-\varphi)e_h. \tag{A3}$$

[63] Since we only consider pure strategies, in equilibrium some A-lenders target AAp-borrowers and some target AAn-borrowers. We need at least two lenders offering each loan, and hence we need at least four lenders in each class, $N \geqslant 4$.

Lemma 1 *If* AAp*-borrowers prefer credit-history building to a safe loan, then so do* AAn*-borrowers.*

Proof. Add $\beta\Pr(e_h|AAn)(e_h - \underline{x} - x_2^p)$ to both sides of (A2):

$$q_h^{AA}\underline{x} + q_h^{AAp}x_2^p + \beta\Pr(e_h|AAn)(e_h - \underline{x} - x_2^p) + \beta\Pr(e_h|AAp)(e_h - \underline{x} - x_2^p)$$
$$\geqslant \bar{q}\varphi e_\ell + \beta\Pr(e_h|AAp)(e_h - \varphi e_\ell) + \beta\Pr(e_h|AAn)(e_h - \underline{x} - x_2^p).$$

Using the AAn's incentive constraint, we have

$$q_h^{AA}\underline{x} + q_h^{AAn}x_2^n + \beta\Pr(e_h|AAn)e_h(1-\varphi) + \beta\Pr(e_h|AAp)(e_h - \underline{x} - x_2^p)$$
$$\geqslant \bar{q}\varphi e_\ell + \beta\Pr(e_h|AAp)(e_h - \varphi e_\ell) + \beta\Pr(e_h|AAn)(e_h - \underline{x} - x_2^p),$$
$$q_h^{AA}\underline{x} + q_h^{AAn}x_2^n + \beta\Pr(e_h|AAn)e_h(1-\varphi)$$
$$\geqslant \bar{q}\varphi e_\ell + \beta\Pr(e_h|AAp)(x_2^p + \underline{x} - \varphi e_\ell) + \beta\Pr(e_h|AAn)(e_h - \underline{x} - x_2^p).$$

Add and subtract $\beta\Pr(e_h|AAn)(e_h - \varphi e_\ell)$ on the right-hand side:

$$q_h^{AA}\underline{x} + q_h^{AAn}x_2^n + \beta\Pr(e_h|AAn)e_h(1-\varphi) \geqslant \bar{q}\varphi e_\ell + \beta\Pr(e_h|AAn)(e_h - \varphi e_\ell)$$
$$+ \beta\Pr(e_h|AAp)(x_2^p + \underline{x} - \varphi e_\ell) + \beta\Pr(e_h|AAn)(e_h - \underline{x} - x_2^p) - \beta\Pr(e_h|AAn)(e_h - \varphi e_\ell),$$

which simplifies to

$$q_h^{AA}\underline{x} + q_h^{AAn}x_2^n + \beta\Pr(e_h|AAn)e_h(1-\varphi) \tag{A4}$$
$$\geqslant \bar{q}\varphi e_\ell + \beta\Pr(e_h|AAn)(e_h - \varphi e_\ell) + \beta(\Pr(e_h|AAp) - \Pr(e_h|AAn))(x_2^p + \underline{x} - \varphi e_\ell).$$

Notice that if AAp-borrowers prefer credit-history building to a safe loan, then $x_2^p + \underline{x} > \varphi e_\ell$. Since $q_h^{AA} < q_h^{AAp} < \bar{q}$, (A2) can be written as

$$q_h^{AAp}\underline{x} + q_h^{AAp}x_2^p + \beta\Pr(e_h|AAp)(e_h - \underline{x} - x_2^p) \geqslant q_h^{AAp}\varphi e_\ell + \beta\Pr(e_h|AAp)(e_h - \varphi e_\ell).$$

Hence $q_h^{AAp}(x_2^p + \underline{x} - \varphi e_\ell) + \beta\Pr(e_h|AAp)(\varphi e_\ell - \underline{x} - x_2^p) \geqslant 0$, or equivalently $(x_2^p + \underline{x} - \varphi e_\ell)\Pr(e_h|AAp)\,(\bar{q} - \beta) \geqslant 0$. Hence, we must have $x_2^p + \underline{x} \geqslant \varphi e_\ell$.

Returning to (A4), since $\Pr(e_h|AAp) > \Pr(e_h|AAn)$ and $x_2^p + \underline{x} \geqslant \varphi e_\ell$ we have that AAn-borrowers strictly prefer their undistorted large loan to the safe loan. □

Proposition 3 *Let* $\beta \in (0, \bar{q})$*. Credit-history building described above is an equilibrium if* (A2) *holds, the fraction* ζ *of borrowers with the* z_p*-state is small enough, and the endowment levels* e_h *and* e_ℓ *are sufficiently far apart.*

Proof. First, we illustrate that (A3) is satisfied if ζ is small enough. Second, we show that for the parameter set such that (A2) and (A3) both hold not to be empty, we need that e_h and e_ℓ are sufficiently far apart.

Rearranging (A3) yields $\beta \Pr(e_h|AAp)(\varphi e_h - \underline{x} - x_2^p) \geqslant q_h^{AA}(\varphi e_h - \underline{x}) - q_h^{AAp} x_2^p$. Adding and subtracting $q_h^{AAp}(\varphi e_h - \underline{x} - x_2^p)$ to the right-hand side yields $\beta \Pr(e_h|AAp)(\varphi e_h - \underline{x} - x_2^p) \geqslant q_h^{AAp}(\varphi e_h - \underline{x} - x_2^p) + (q_h^{AA} - q_h^{AAp})(\varphi e_h - \underline{x})$, which can be rewritten as

$$(q_h^{AAp} - q_h^{AA})(\varphi e_h - \underline{x}) \geqslant (q_h^{AAp} - \beta \Pr(e_h|AAp))(\varphi e_h - \underline{x} - x_2^p), \tag{A5}$$

or

$$\Pr(e_h|AAp) - \Pr(e_h|AA) \geqslant \frac{(\bar{q} - \beta) \Pr(e_h|AAp)(\varphi e_h - \underline{x} - x_2^p)}{\bar{q}(\varphi e_h - \underline{x})}.$$

This condition further simplifies to

$$(1 - \zeta) \left[\Pr(e_h|AAp) - \Pr(e_h|AAn)\right] \geqslant \frac{(\bar{q} - \beta) \Pr(e_h|AAp)(\varphi e_h - \underline{x} - x_2^p)}{\bar{q}(\varphi e_h - \underline{x})}.$$

This illustrates that we need ζ to be small enough for the incentive constraint (A3) to hold (since ζ affect the left- but not the right-hand side.)

Further, rewrite (A2) as $q_h^{AA}\underline{x} + q_h^{AAp} x_2^p \geqslant \bar{q}\varphi e_\ell + \beta \Pr(e_h|AAp)(x_2^p + \underline{x} - \varphi e_\ell)$. Adding and subtracting $q_h^{AAp}(x_2^p + \underline{x} - \varphi e_\ell)$ on the left-hand side we get

$$\begin{aligned}
&(q_h^{AAp} - \beta \Pr(e_h|AAp))(x_2^p + \underline{x} - \varphi e_\ell) \geqslant \bar{q}\varphi e_\ell - q_h^{AA}\underline{x} + q_h^{AAp}(\underline{x} - \varphi e_\ell),\\
&[q_h^{AAp} - \beta \Pr(e_h|AAp)](x_2^p + \underline{x} - \varphi e_\ell)\\
&\geqslant \bar{q}(1 - \Pr(e_h|AAp))\varphi e_\ell + \bar{q}(1 - \zeta)\left[\Pr(e_h|AAp) - \Pr(e_\ell|AA, z_n)\right]\underline{x}.
\end{aligned}$$

For (A3), or its equivalent (A5), to hold we need

$$(q_h^{AAp} - q_h^{AA})\frac{\varphi e_h - \underline{x}}{\varphi e_h - \underline{x} - x_2^p} \geqslant q_h^{AAp} - \beta \Pr(e_h|AAp).$$

For (A2) to hold, we need

$$q_h^{AAp} - \beta \Pr(e_h|AAp) \geqslant \frac{(\bar{q} - q_h^{AAp})\varphi e_\ell + (q_h^{AAp} - q_h^{AA})\underline{x}}{x_2^p + \underline{x} - \varphi e_\ell}.$$

For the intersection of the two sets defined by these conditions to be non-empty, we need

$$(q_h^{AAp} - q_h^{AA})\frac{\varphi e_h - \underline{x}}{\varphi e_h - \underline{x} - x_2^p} \geqslant \frac{(\bar{q} - q_h^{AAp})\varphi e_\ell + (q_h^{AAp} - q_h^{AA})\underline{x}}{x_2^p + \underline{x} - \varphi e_\ell}.$$

Further manipulation yields

$$\begin{aligned}
&(q_h^{AAp} - q_h^{AA})\left(x_2^p(\varphi e_h - \underline{x}) + \underline{x}(\varphi e_h - \underline{x}) - \varphi e_\ell(\varphi e_h - \underline{x})\right) \\
&\geqslant (\bar{q} - q_h^{AAp})\varphi e_\ell(\varphi e_h - \underline{x} - x_2^p) + (q_h^{AAp} - q_h^{AA})\underline{x}(\varphi e_h - \underline{x} - x_2^p), \\
&(q_h^{AAp} - q_h^{AA})\left(x_2^p \varphi e_h - \varphi e_\ell(\varphi e_h - \underline{x})\right) \geqslant (\bar{q} - q_h^{AAp})\varphi e_\ell(\varphi e_h - \underline{x} - x_2^p), \\
&x_2^p \varphi e_h(q_h^{AAp} - q_h^{AA}) - \varphi^2 e_\ell e_h\left(\bar{q} - q_h^{AA}\right) + \varphi e_\ell \underline{x}\left(\bar{q} - q_h^{AA}\right) + x_2^p \varphi e_\ell(\bar{q} - q_h^{AAp}) \geqslant 0, \\
&x_2^p\left[e_h(q_h^{AAp} - q_h^{AA}) + e_\ell(\bar{q} - q_h^{AAp})\right] \geqslant e_\ell(\bar{q} - q_h^{AA})\left[\varphi e_h - \underline{x}\right].
\end{aligned}$$

Using (A1), this is equivalent to

$$\frac{\bar{q}\Pr(e_h|AAn) - \beta\Pr(e_h|AAn)}{\bar{q}\Pr(e_h|AAp) - \beta\Pr(e_h|AAn)}\left[e_h\frac{\Pr(e_h|AAp) - \Pr(e_h|AA)}{1 - \Pr(e_h|AA)} + e_\ell\right] \geqslant e_\ell,$$

which is satisfied if e_h is sufficiently different from e_ℓ. □

E Agents' Problems

In order to facilitate characterization of equilibria, we define the sequence of problems faced by each agent in the order implied by backward induction. In the middle of stage 2, after lenders have made their stage-2 offers, the borrower has observed two sets of offers, O_1 and O_2, and her own credit history $h_2^P = (x_1, q_1, j_1)$. Let $h_2^B = (O_1, h_2^P, O_2)$ denote this information set of the borrower. The borrower's stage-2 action is to choose an offer from O_2 (or possibly reject all offers). She does so based in part on her posterior beliefs about her own quality state induced by the history (and her understanding of lenders' strategies). We denote $\theta_2^B(e|h_2^B)$ the probability the borrower assigns in stage 2 to receiving endowment e in the second period. Note that this probability is a convolution of the posterior belief of the borrower regarding her underlying quality s and the probability distribution over outcomes implied by this quality. Of course, the borrower forms her posterior about her underlying quality based on public and private histories, as well as her understanding of lenders' equilibrium strategies—on the equilibrium path, it is obtained using Bayes' rule. The borrower's stage-2 action maximizes her expected payoff under θ_2^B and so solves

$$V_2\left(h_2^B\right) = \max_{(x_2,q_2,j)\in O_2\bigcup\{(0,0,0)\}} u(q_1x_1+q_2x_2)+\beta\sum_e \theta_2^B(e|h_2^B)u(\max\{e-x_1-x_2,(1-\varphi)e\}).$$

At the beginning of stage 2, everyone has observed the public credit history of the borrower $h_2^P = (x_1, q_1, j_1)$. Additionally, each lender k knows his private signal about the borrower's state, σ_k, and his offer to the borrower in the first stage, (x_1^k, q_1^k). Thus, the private history of the lender k is $h_2^k = (h_2^P, \sigma_k, (x_1^k, q_1^k))$. When choosing his second-stage offer, the kth lender forms expectations of other lenders' offers. Similar to the borrower, the lender forms his posterior belief $\mu_2(\sigma_{-k})$ regarding the other class' signal based in part on his understanding of equilibrium strategies. Equilibrium strategies imply a mapping from the vector of realized signals and the observed public history into an offer set O_2, which will be faced by the borrower. For any (x, q) offered by the kth lender, denote by ξ_2^k the probability of that offer being accepted (as perceived by the kth lender given the equilibrium strategies of the borrower and the other lenders).[64] Then, the optimal offer made by lender k solves the following maximization problem:

$$W_2^k\left(h_2^k\right) = \max_{(x,q)}\sum_{\sigma_{-k}}\mu_2(\sigma_{-k}|h_2^k)\,\xi_2^k\,(x,q)$$
$$\times\left[-qx-q_1x_1\mathbb{1}_{j_1=k}+\bar{q}\,\left(x+x_1\mathbb{1}_{j_1=k}\right)\sum_e\theta_2^L(e|h_2^k,j_2=k)\mathbb{1}_{[\varphi e\geqslant x_1+x]}\right],$$

where $\theta_2^L(e|\cdot)$ is the lender's posterior probability that the borrower will receive endowment e conditional on the lender's information at the beginning of stage 2 and the fact that her offer was accepted by the borrower.

In stage 1, the borrower chooses among offers in the set O_1 (and the option of rejecting all offers) to maximize

$$V_1(O_1) = \max_{(x,q,k)\in O_1\bigcup\{(0,0,0)\}}\mathbb{E}V_2(O_1,(x,q,k),O_2(x,q,k)).$$

Note that the borrower understands that her choice of (x, q) influences not only her payoffs in V_2 directly but also the set of offers she will receive in stage 2, O_2.

Similarly, lenders in stage 1 understand that the offer they make, if accepted, may

[64] To be more precise, $\xi_2^k = \xi_2^k\left((x,q,k)|(x_1^k,q_1^k,k),O_1^{-k}(\sigma_k,\sigma_{-k}),h_2^p,O_2^{-k}(\sigma_k,\sigma_{-k},h_2^P)\right)$, where σ_k is the signal observed by the k-th lender, σ_{-k} is the signal observed by lenders of the other class, and O_i^{-k} is the offer set excluding the offer made by the k-th lender in stage $i = 1, 2$.

influence the posteriors of other lenders in the second stage.[65] Having observed their signal, they make an offer that maximizes their expected profits:

$$W_1^k(\sigma_k) = \max_{(x,q)} \sum_{\sigma_{-k}} \mu_1(\sigma_{-k}|\sigma_k) \quad \Big[\xi_1^k(x,q)\, W_2^k((x,q,k),\sigma_k,(x,q)) \\ + \left(1-\xi_1^k(x,q)\right) W_2^k((x_{-k},q_{-k},-k),\sigma_k,(x,q))\Big],$$

where ξ_1^k and θ_1^L are defined similar to their stage-2 counterparts. Note that, if accepted, the lender's offer influences her payoffs not only directly but also by affecting the offer set O_2 in the subsequent stage.

F Proofs and Equilibrium Constructions for the General Model

F.1 Preliminaries

The following expressions will be useful for our equilibrium construction throughout the rest of this appendix. We have

$$\begin{aligned}
&\Pr(g|A) = \frac{\alpha(1+\rho)}{\alpha(1+\rho)+(1-\alpha)(1-\rho)},\ \Pr(b|B) = \frac{(1-\alpha)(1+\rho)}{\alpha(1-\rho)+(1-\alpha)(1+\rho)},\\
&\Pr(g|B) = \frac{\alpha(1-\rho)}{\alpha(1-\rho)+(1-\alpha)(1+\rho)},\ \Pr(b|A) = \frac{(1-\alpha)(1-\rho)}{\alpha(1+\rho)+(1-\alpha)(1-\rho)},\\
&\Pr(g|AA) = \frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2},\ \Pr(b|BB) = \frac{(1-\alpha)(1+\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2},\\
&\Pr(g|AB) = \alpha,\ \Pr(b|AB) = 1-\alpha,\\
&\Pr(g|BB) = \frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2},\ \Pr(b|AA) = \frac{(1-\alpha)(1-\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}.
\end{aligned}$$

[65] In our setting, an individual lender's deviation does not change the borrower's posterior, since the borrower is facing many lenders. However, it may affect other lenders' posterior, since lenders do not observe the offer set O_1, only the borrower's choice from that set.

In addition,

$$\Pr(AA|A) = \frac{1}{2}\frac{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2}{\alpha(1+\rho)+(1-\alpha)(1-\rho)}, \tag{A6}$$

$$\Pr(AB|A) = \frac{1}{2}\frac{(1+\rho)(1-\rho)}{\alpha(1+\rho)+(1-\alpha)(1-\rho)}. \tag{A7}$$

Moreover,

$$\Pr(\text{repay } \varphi e_h|AA) = \delta \Pr(g|AA), \quad \Pr(\text{repay } \varphi e_m|AA) = 1 - \delta \Pr(b|AA),$$
$$\Pr(\text{repay } \varphi e_h|AB) = \delta \Pr(g|AB), \quad \Pr(\text{repay } \varphi e_m|AB) = 1 - \delta \Pr(b|AB).$$

Recall that we restrict stage-1 offers to φe_1 for $e_1 \in E$, and given a history φe_1, we restrict stage-2 offers to $\varphi(e_2 - e_1)$ for $e_2 \in E$ and $e_2 > e_1$.

F.2 Symmetric-Information Outcomes

To establish a benchmark for our analysis, consider a variant of our model environment in which all lenders' signals are public information. The multi-stage nature of period I is irrelevant in this setting, as there is no need to aggregate any information. We can thus simply restrict attention to equilibria where all borrowing occurs in the last stage of the period, which avoids any concerns of debt dilution. All loans are then competitively priced, and we can simply think of the borrowers as choosing their preferred loan size, given actuarially fair interest rates appropriate for the specific type of the borrower.

All of the equilibrium examples in the paper share one key feature of the symmetric-information benchmark. Namely, the equilibrium outcome in the limiting case as ρ approaches 1 features full separation in loan sizes between the three borrower types. I.e., for ρ arbitrarily close to 1, BB-borrowers take on a small loan, AB-borrowers choose a medium loan, and AA-borrowers get a large loan in the equilibrium of the symmetric-information environment. The restrictions on the parameter values that yield this outcome, which we will sometimes refer to as lmh, are as follows.

Assumption 1 *Assume that parameter values satisfy the following conditions:*

(i) $(1-\delta(1-\alpha))\, e_m > e_\ell$*, or, equivalently,* $q_m^{AB} e_m > \bar{q} e_\ell$*;*

(ii) $(1-\delta(1-\alpha))\, e_m > \delta\alpha e_h$*, or, equivalently,* $q_m^{AB} e_m > q_h^{AB} e_h$*;*

(iii) $\delta e_h > e_m$*;*

(iv) $e_\ell > (1-\delta)e_m$.

We explain both formally and intuitively why these conditions imply the $\ell m h$ outcome in the proof of the following proposition.

Proposition 4 *If parameter values satisfy Assumption 1, then the symmetric-information equilibrium outcome is:*

(i) for ρ arbitrarily close to 1, BB-borrowers get $(\varphi e_\ell, \bar{q})$, AB-borrowers get $(\varphi e_m, q_m^{AB})$, and AA-borrowers get $(\varphi e_h, q_h^{AA})$;

(ii) for $\rho = 0$, all borrowers receive a medium-size loan.

Proof. First, note that the symmetric structure of the signals is such that the posterior regarding the underlying state of AB-borrowers is the same as the uninformed prior and thus does not depend on the precision of the signal. Hence, parts (i) and (ii) of Assumption 1 guarantee that AB-borrowers choose the medium-size loan under actuarially fair loan pricing. But these same conditions then guarantee that all borrowers choose medium-size loans when signals are completely uninformative.

Assumption 1 (iii) guarantees that, when signals are perfectly informative, AA-borrowers take on a large loan if all prices are actuarially fair. Assumption 1 (iv) guarantees that BB-borrowers in this situation choose the small loan.

Note that this set of conditions also ensures that AA-borrowers do not choose the small loan. To see this, note that the condition for AA to prefer a medium loan to a small one is $[1-\delta \Pr(b|AA)]e_m > e_\ell$. Note that $\Pr(b|AA) < 1-\alpha$ whenever $\rho > 0$. Hence Assumption 1 (i) ensures that AA-borrowers prefer a medium loan to a small one. □

Corollary 1 *If parameter values satisfy Assumption 1, then BB-borrower prefers a medium-size loan to a large loan if both loans are priced actuarially fairly. I.e., $q_m^{BB}e_m \geqslant q_h^{BB}e_h$.*

Proof. By Assumption 1 (ii), $e_m/e_h \geqslant q_h^{AB}/q_m^{AB}$. In order to establish our claim, we need to show that $q_h^{AB}/q_m^{AB} \geqslant q_h^{BB}/q_m^{BB}$, which we do by establishing $q_m^{BB}/q_m^{AB} \geqslant q_h^{BB}/q_h^{AB}$. The actuarially fair prices are

$$
\begin{aligned}
q_m^{AB} &= \bar{q}\left(1-\delta(1-\alpha)\right), \\
q_h^{AB} &= \bar{q}\delta\alpha, \\
q_m^{BB} &= \bar{q}\left[1-\delta\frac{(1-\alpha)(1+\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}\right], \\
q_h^{BB} &= \bar{q}\delta\frac{\alpha(1-\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}.
\end{aligned}
$$

Plugging these in, we have

$$\begin{aligned}\frac{q_m^{BB}}{q_m^{AB}} - \frac{q_h^{BB}}{q_h^{AB}} &= \frac{1}{1-\delta(1-\alpha)}\left[1-\delta\frac{(1-\alpha)(1+\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}\right] - \frac{(1-\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}\\ &= \frac{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2-\delta(1-\alpha)(1+\rho)^2-(1-\delta(1-\alpha))(1-\rho)^2}{(1-\delta(1-\alpha))((1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2)}\\ &= \frac{(1-\delta)(1-\alpha)(1+\rho)^2-(1-\delta)(1-\alpha)(1-\rho)^2}{(1-\delta(1-\alpha))((1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2)} \geqslant 0.\end{aligned}$$

□

Proposition 5 *Suppose parameter values satisfy Assumption 1. Then*

(i) there exists $\rho^{BB} \in (0,1)$ such that BB-borrowers take on $(\varphi e_\ell, \bar{q})$ in the symmetric-information equilibrium whenever $\rho > \rho^{BB}$, and they choose $(\varphi e_m, q_m^{BB})$ whenever $\rho < \rho^{BB}$;

(ii) there exists $\rho^{AA} \in (0,1)$ such that AA-borrowers take on $(\varphi e_h, q_h^{AA})$ in the symmetric-information equilibrium whenever $\rho > \rho^{AA}$, and they choose $(\varphi e_m, q_m^{AA})$ whenever $\rho < \rho^{AA}$.

Proof. Since borrowers are impatient, they simply maximize the size of the loan advance they receive in period I. The medium-size loan yields $\varphi e_m q_m^\omega$ to a type-ω borrower, where $q_m^\omega = \bar{q}\,(1-\delta\,\Pr(b|\omega))$.

(i) Note that $\Pr(b|BB)$ is increasing in ρ, and thus q_m^{BB} is monotonically decreasing in ρ. On the other hand, the advance on the safe loan $(\varphi e_\ell, \bar{q})$ is not affected by ρ. Since the medium-size loan is preferred by BB-borrowers when $\rho = 0$, and the small loan is preferred when $\rho = 1$ (as was established in Proposition 4), there must be an interior ρ^{BB}, as described in the statement of this proposition.

(ii) The advance on the large loan is given by $\varphi e_h q_h^\omega$, where $q_h^\omega = \bar{q}\delta\Pr(g|\omega)$. Just like in the case above, it is straightforward to show that $\varphi e_h q_h^{AA} - \varphi e_m q_m^{AA}$ is strictly increasing in ρ. And since the advance to an AA-borrower from a large loan is greater than that from a medium-size loan when $\rho = 1$, and since the opposite is true when $\rho = 0$ (both premises are guaranteed by Proposition 4), there must exist an interior ρ^{AA} described in the statement of this proposition. Making this argument more explicit, the large loan yields a (weakly) larger loan advance whenever

$$\begin{aligned}0 \leqslant q_h^{AA} e_h - q_m^{AA} e_m &= \bar{q}\delta \Pr(g|AA) e_h - \bar{q}[\Pr(g|AA) + (1-\delta)(1-\Pr(g|AA)] e_m \\ &= \bar{q}\frac{\delta\alpha(1+\rho)^2 e_h - \alpha(1+\rho)^2 e_m - (1-\delta)(1-\alpha)(1-\rho)^2 e_m}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} \\ &= \bar{q}\frac{\alpha(\delta e_h - e_m)y - (1-\delta)(1-\alpha)e_m}{\alpha y + 1 - \alpha}, \end{aligned} \tag{A8}$$

where $y = (1+\rho)^2/(1-\rho)^2$ is strictly increasing in ρ. The derivative of the right-hand side of (A8) with respect to y is

$$\bar{q}\alpha(1-\alpha)\frac{(\delta e_h - e_m) + (1-\delta)e_m}{(\alpha y + 1 - \alpha)^2} = \bar{q}\alpha(1-\alpha)\delta\frac{e_h - e_m}{(\alpha y + 1 - \alpha)^2} > 0.$$

Thus $q_h^{AA} e_h - q_m^{AA} e_m$ is strictly increasing in ρ, implying that there is a unique root between 0 and 1. We denote this root by ρ_{AA}. □

F.3 Equilibrium Outcome 1: $\mathtt{lmh}$ without Cross-Subsidization

We construct the equilibrium as follows.

F.3.1 On-Path Actions

Stage 1:

- B-lenders make no offers;
- A-lenders offer $\left(\varphi e_\ell, q_h^{AA}\right)$;
- Only borrowers with two such offers (AA-borrowers) accept one.

Stage 2:

- A-lenders whose offer was not accepted, but observe an accepted offer from the *opposite* class, learn that the borrower is AA and offer $\left(\varphi\left(e_h - e_\ell\right), q_h^{AA}\right)$. Such an offer is accepted by AA-borrowers.
- A-lenders whose offer was not accepted, but who observe an accepted offer from *their* class, offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$. (Note that on the equilibrium path, this offer is not accepted by any borrowers. However, we specify this offer in order to ensure that AB-borrowers do not mimic AA-borrowers.)

– A-lenders who observe no accepted offer learn that the borrower is AB, and offer $\left(\varphi e_m, q_m^{AB}\right)$. This offer is accepted by the AB-borrowers.

– B-lenders, who *never* observe an accepted offer in stage 1 (on-path), offer $(\varphi e_\ell, \overline{q})$. This offer is accepted by the BB-borrowers.

Before proceeding, note a couple of things about this equilibrium. First, this equilibrium is symmetric: lenders' offers are a function of their signal and public information only, so we forego class identifiers. Second, on path, this equilibrium features full information for the borrower after stage 1. By observing the number of offers that she receives in stage 1, a borrower is certain whether she is AA (offers from all lenders), AB (offers from only one class of lenders), or BB (no offers).

F.3.2 Equilibrium Payoffs

The payoffs to borrowers in equilibrium are as follows:

- AA-borrowers: $\varphi e_\ell q_h^{AA} + \varphi\left(e_h - e_\ell\right) q_h^{AA} = \varphi e_h q_h^{AA}$;
- AB-borrowers: $\varphi e_m q_m^{AB}$;
- BB-borrowers: $\varphi e_\ell \overline{q}$.

F.3.3 Equilibrium Conditions

Before we proceed with construction of beliefs and (off-path) strategies, we state necessary conditions on the model parameters so that our constructed equilibrium candidate is indeed an equilibrium. We later show that these conditions together with Assumption 1 are sufficient to ensure that relevant incentive constraints are satisfied.

Condition 1 *Suppose that the model parameters satisfy*

(i)

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} \leqslant 1 - \delta(1-\alpha),$$

or, equivalently, $q_h^{AA} \leqslant q_m^{AB}$; *this ensures that the* AB*-borrowers do not accept a small loan in stage 1.*

(ii)

$$[1-\delta(1-\alpha)](e_m - e_\ell) \geqslant \delta\alpha(e_h - e_\ell),$$

or, equivalently, $q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell)$*; this ensures that upon acceptance of a small loan in stage 1, prices are such that an* AB*-borrower is better off being topped up to a medium loan rather than a large one. (Note that this condition implies that* $q_m^{AB} e_m > q_h^{AB} e_h$*, which is Assumption 1(ii).)*

(iii)

$$\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2}(e_h - e_\ell) \geqslant \left[1 - \delta \frac{(1-\alpha)(1-\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right](e_m - e_\ell),$$

or, equivalently, $q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell)$*; this ensures that upon acceptance of a small loan in stage 1, prices are such that an* AA*-borrower is better off being topped up to a large loan rather than a medium one.*

(iv)

$$e_\ell \geqslant \left(1 - \delta \frac{(1-\alpha)(1+\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right) e_m,$$

or, equivalently, $\bar{q} e_\ell \geqslant q_m^{BB} e_m$*. Note that by Assumption 1 (ii) and Corollary 1 we also have* $q_m^{BB} e_m \geqslant q_h^{BB} e_h$*. Combining, we have* $\bar{q} e_\ell \geqslant \max\left\{q_m^{BB} e_m, q_h^{BB} e_h\right\}$*. Thus, the imposed condition ensures that* BB*-borrowers prefer to take on a small loan at the risk-free price, rather than a medium or large loan at the actuarially-fair price reflecting their risk.*

(v)

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} e_h \geqslant \left[1 - \delta(1-\alpha)\right] e_m,$$

or, equivalently, $q_h^{AA} e_h \geqslant q_m^{AB} e_m$*; this ensures that* AA*-borrowers are better off accepting their stage-1 offer.*

F.3.4 Beliefs

We classify out-of-equilibrium histories and beliefs based on the size of the stage-1 loan. In the interest of brevity, for the rest of this appendix when describing beliefs and strategies, we focus on only three possible loan sizes, the ones that may occur as equilibrium outcomes: $\{\varphi e_\ell, \varphi e_m, \varphi e_h\}$. The treatment of intermediate loan sizes is a straightforward extension of the construction presented below: loans with $x \in (\varphi e_\ell, \varphi e_m)$ are treated just like medium loans, and loans with $x \in (\varphi e_m, \varphi e_h)$ are treated just like large loans, except they are, of course, topped up to a large loan in the second stage (note that loans smaller than φe_ℓ are not possible). This generalization would require that all of the pricing thresholds $\hat{q}$ below are explicitly made functions of the loan size x. The details of this generalization are available from the authors upon request.

1. *Small Loans:* Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$.

    - Beliefs of A-lenders when the loan came from *the opposite* class are

$$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

    - Beliefs of A-lenders when the loan came from *their* class are

$$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

    - Beliefs of B-lenders when the loan came from *the opposite* class are

$$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < \max\{q_h^{AA}, \hat{q}_\ell\}, \\ 1 & q \geqslant \max\{q_h^{AA}, \hat{q}_\ell\}, \end{cases}$$

    where $\hat{q}_\ell e_\ell + q_m^{BB}\left(e_m - e_\ell\right) = \bar{q} e_\ell$;

    - Beliefs of B-lenders when the loan came from *their* class are

$$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}. \end{cases}$$

2. *Medium Loans:* Suppose the borrower has accepted a loan $(\varphi e_m, q)$.

    - Beliefs of A-lenders when the loan came from the opposite class are

$$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

    - Beliefs of A-lenders when the loan came from their class are

$$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

- Beliefs of B-lenders when the loan came from the opposite class are

$$\Pr\left(\sigma^{-}=A\right)=\begin{cases}0 & q<\max\{\hat{q}_{m2},q_h^{AA}\},\\ 1 & q\geqslant\max\{\hat{q}_{m2},q_h^{AA}\};\end{cases}$$

  where $\hat{q}_{m2}e_m+q_h^{BB}(e_h-e_m)=\bar{q}e_\ell$.

- Beliefs of B-lenders when the loan came from their class are

$$\Pr\left(\sigma^{-}=A\right)=\begin{cases}0 & q<q_h^{AA},\\ 1 & q\geqslant q_h^{AA}.\end{cases}$$

3. *Large Loans:* Suppose the borrower has accepted a loan $(\varphi e_h, q)$. Then lenders' beliefs in this scenario going forward are irrelevant.

4. *No loans*: All lenders believe $\Pr(\sigma^{-}=A)=0$.

F.3.5 Strategies

Borrowers' (off-path) Strategies in Stage 1 Strategies of borrowers upon observing offer(s) in the first stage:[66]

- AA-borrowers: Suppose a borrower observes at least $2N-1$ offers of $\left(\varphi e_\ell, q_h^{AA}\right)$.
  - Small loan: if one lender offers $(\varphi e_\ell, q)$ with $q\neq q_h^{AA}$, the borrower accepts that offer if and only if $q>q_h^{AA}$.
  - Medium loan: if one lender offers $(\varphi e_m, q)$, the borrower accepts if and only if $q>q_h^{AA}$.
  - Large loan: if one lender offers $(\varphi e_h, q)$, the borrower accepts if and only if $q>q_h^{AA}$.
- AB-borrowers with N offers: Suppose that the borrower receives $N-1$ offers of $\left(\varphi e_\ell, q_h^{AA}\right)$; that is, no B-lenders make offers, but one A-lender offers something off path.
  - Small loan: if one lender offers $(\varphi e_\ell, q)$ with $q\neq q_h^{AA}$, the borrower accepts if $q>q_m^{AB}$.

[66] The list below is restricted to histories with a single deviation.

– Medium loan: if one lender offers $(\varphi e_m, q)$, the borrower accepts if $q > \hat{q}_{m1}$ where

$$\hat{q}_{m1} e_m + q_h^{AB}(e_h - e_m) = q_m^{AB} e_m.$$

– Large loan: if one lender offers $(\varphi e_h, q)$, the borrower accepts if $q e_h \geqslant q_m^{AB} e_m$.

- AB-borrowers with $N+1$ offers: Suppose that the borrower receives N offers of $\left(\varphi e_\ell, q_h^{AA}\right)$ and one additional offer; that is, one B-lender made an offer.
  – Small loan: if the deviating B-lender offers $(\varphi e_\ell, q)$, the borrower accepts if and only if $q \geqslant q_h^{AA}$.
  – Medium loan: if the deviating B-lender offers $(\varphi e_m, q)$, the borrower accepts if and only if $q \geqslant \min\{\hat{q}_{m1}, q_h^{AA}\}$.
  – Large loan: if the deviating B-lender offers $(\varphi e_h, q)$, the borrower accepts if $q e_h \geqslant q_m^{AB} e_m$.
- BB-borrowers: Suppose that a borrower observes just one offer.
  – Small loan: if the deviating B-lender offers $(\varphi e_\ell, q)$, the borrower accepts if and only if $q \geqslant \hat{q}_\ell$.
  – Medium loan: if the one lender offers $(\varphi e_m, q)$, the borrower accepts if $q \geqslant \hat{q}_{m2}$.
  – Large loan: if the one lender offers $(\varphi e_h, q)$, the borrower accepts if $q e_h \geqslant \bar{q} e_\ell$.

Lenders' Strategies in Stage 2 We next describe lenders' strategies for any credit history in stage 2 (i.e. any information set of lenders in stage 2).

1. *Small loan* $(\varphi e_\ell, q)$ from stage 1
   - If the first-stage loan came from the other class of lenders, then A-lenders
     – offer $\left(\varphi\left(e_h - e_\ell\right), q_h^{AA}\right)$ if $q \geqslant q_h^{AA}$,
     – offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$ if $q < q_h^{AA}$.
   - If the first-stage loan came from their class of lenders, then A-lenders offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$.
   - If the first-stage loan came from the other class of lenders, then B-lenders
     – offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$ if $q \geqslant \max\{q_h^{AA}, \hat{q}_\ell\}$,
     – offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{BB}\right)$ if $q < \max\{q_h^{AA}, \hat{q}_\ell\}$.

- If the first-stage loan came from their class of lenders, then B-lenders offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{BB}\right)$.

2. *Medium loan* $(\varphi e_m, q)$ from stage 1

   - If the first-stage loan came from the other class of lenders, then A-lenders
     – offer $\left(\varphi\left(e_h - e_m\right), q_h^{AA}\right)$ if $q \geqslant q_h^{AA}$,
     – offer $\left(\varphi\left(e_h - e_m\right), q_h^{AB}\right)$ if $q < q_h^{AA}$.
   - If the first-stage loan came from their class of lenders, then A-lenders offer $\left(\varphi\left(e_h - e_m\right), q_h^{AB}\right)$.
   - If the first-stage loan came from the other class of lenders, then B-lenders
     – offer $\left(\varphi\left(e_h - e_m\right), q_h^{AB}\right)$ if $q \geqslant q_h^{AA}$,
     – offer $\left(\varphi\left(e_h - e_m\right), q_h^{BB}\right)$ if $q < q_h^{AA}$.
   - If the first-stage loan came from their class of lenders, then B-lenders offer $\left(\varphi\left(e_h - e_m\right), q_h^{BB}\right)$.

3. *Large loan* from stage 1

   - Lenders make no offers in stage 2 if they see a large loan from stage 1.

4. *No loan* in stage 1

   - A-lenders offer $\left(\varphi e_m, q_m^{AB}\right)$.
   - B-lenders offer $(\varphi e_\ell, \bar{q})$.

F.3.6 Incentives

We now verify that given Assumption 1 and Condition 1, the strategies and beliefs described above constitute an equilibrium.

Borrowers' Stage 1 Deviations. Consider first possible deviations by borrowers in stage 1.

1. An AA-borrower could reject both stage 1 offers. Accepting is optimal as long as

$$q_h^{AA} e_h \geqslant q_m^{AB} e_m, \tag{A9}$$

   which is ensured by Condition 1 (v).

2. An AB-borrower could accept a stage 1 offer. Rejecting is optimal as long as

$$q_m^{AB} e_m \geqslant q_h^{AA} e_\ell + q_m^{AB}(e_m - e_\ell), \tag{A10}$$

or $q_m^{AB} \geqslant q_h^{AA}$, which is ensured by Condition 1 (i).

Lenders' Stage 1 Deviations. Since we have already specified the borrowers' and lenders' strategies following stage-1 deviation offers, all that remains is to verify that it is not optimal for lenders to deviate in stage 1.

- A-lenders: do they want to offer anything other than $\left(\varphi e_\ell, q_h^{AA}\right)$?

  1. *Small loans*
     - An offer $(\varphi e_\ell, q)$ with $q < q_h^{AA}$ will not be accepted by anyone.
     - An offer $(\varphi e_\ell, q)$ with $q_h^{AA} < q < q_m^{AB}$ will be accepted only by AA borrowers, who will then be topped up to a large loan in stage 2, making this loan an expected loser.
     - An offer $(\varphi e_\ell, q)$ with $q > q_m^{AB}$ is accepted by AA and AB types, and thus is of course an expected loser. The AA types will be topped up to a large loan, and the AB types to a medium.
  2. *Medium loans*
     - An offer $(\varphi e_\ell, q)$ with $q < \min\left\{q_h^{AA}, \hat{q}_{m1}\right\}$ is not accepted by anyone. An AA borrower would obtain payoff $q\varphi e_m + q_h^{AB}\varphi(e_h - e_m)$ which is smaller than her equilibrium payoff. An AB borrower would obtain $q\varphi e_m + q_h^{AB}(e_h - e_m)$ which, given the definition of $\hat{q}_{m1}$, is smaller than her equilibrium payoff as well.
     - An offer with $\min\left\{\hat{q}_{m1}, q_h^{AA}\right\} < q < \max\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ is accepted by only AB borrowers (if $\hat{q}_{m1} < q_h^{AA}$) or only by AA borrowers (if $q_h^{AA} < \hat{q}_{m1}$). In either case, the accepted offer yields negative expected profits for the lender. If only AB borrowers accept, the lender expects to earn $-q + q_h^{AB}$ (per dollar of face value). From the definition of $\hat{q}_{m1}$, $(\hat{q}_{m1} - q_h^{AB})e_m = q_m^{AB}e_m - q_h^{AB}e_h \geqslant 0$ where the inequality follows from Condition 1 (ii). Hence, $q \geqslant q_h^{AB}$ so the offer yields negative expected profits. If only AA borrowers accept, the lender expects to earn $-q + q_h^{AA}$ (per dollar of face value), which necessarily earns negative expected profits.

- An offer with $q \geqslant \max\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ attracts both AA and AB borrowers and so necessarily loses money on both AA and AB borrowers using the previous argument.

3. *Large loans*
   - An offer with $(\varphi e_h, q)$ is accepted by AA borrowers if and only if $q \geqslant q_h^{AA}$ in which case the offer loses money from AA borrowers.
   - An offer with $(\varphi e_h, q)$ is accepted by AB borrowers if and only if $q e_h \geqslant q_m^{AB} e_m$. Condition 1 (ii) implies $q_m^{AB} e_m \geqslant q_h^{AB} e_h$ so that $q \geqslant q_h^{AB}$. As a result, the offer loses money from AB borrowers.
   - Since any offer $(\varphi e_h, q)$ that is accepted loses money on all types that accept it, A lenders cannot profit by offering large loans in stage 1.

- B-lenders: do they want to offer anything in stage 1?

1. *Small loans*
   - A loan with $q < \min\left\{\hat{q}_\ell, q_h^{AA}\right\}$ is not accepted by anyone.
   - An offer with $\min\{q_h^{AA}, \hat{q}_\ell\} < q < \max\{q_h^{AA}, \hat{q}_\ell\}$ is accepted by only BB borrowers (if $\hat{q}_\ell < q_h^{AA}$) or only by AB borrowers (if $q_h^{AA} < \hat{q}_\ell$). In either case the accepted offer yields negative profits. If only BB borrowers accept, the lender expects to earn $-q + q_m^{BB}$ (per dollar of face value). Using Condition 1 (iv), the definition of $\hat{q}_\ell$ implies $\hat{q}_\ell \geqslant q_m^{BB}$, so the offer is unprofitable. If only AB borrowers accept, the lender expects to earn $-q + q_h^{AB}$ but $q \geqslant q_h^{AA} \geqslant q_h^{AB}$ so the offer is unprofitable.
   - An offer with $q \geqslant \max\{q_h^{AA}, \hat{q}_\ell\}$ is accepted by both AB and BB borrowers and so necessarily loses money on both AA and AB borrowers using the previous argument.

2. *Medium loans*
   - A loan with $q < \hat{q}_{m2}$ and $q < \min\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ is not accepted by any borrowers.
   - A loan with $q \geqslant \hat{q}_{m2}$ is accepted by BB borrowers and necessarily loses money on BB borrowers. Expected profits (per dollar face value) is $-q + q_h^{BB}$. From the definition of $\hat{q}_{m2}$, $(\hat{q}_{m2} - q_h^{BB}) e_m = \bar{q} e_\ell - q_h^{BB} e_h$. Condition 1 (iv) implies $\bar{q} e_\ell \geqslant q_h^{BB} e_h$ and hence this loan loses money.
   - A loan with $q \geqslant \min\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ is accepted by AB borrowers and necessarily loses money on AB borrowers. Expected profits (per dollar face

value) is $-q + q_h^{AB}$. The definition of $\hat{q}_{m1}$ and Condition 1 (ii) immediately implies $\hat{q}_{m1} > q_h^{AB}$. Since $q_h^{AA} \geqslant q_h^{AB}$, it follows that $q_h^{AB} \leqslant \min\{\hat{q}_{m1}, q_h^{AA}\}$ so the offer necessarily loses money.

3. Large loans

   – An offer with $(\varphi e_h, q)$ is accepted by AB borrowers if and only if $qe_h \geqslant q_m^{AB}$. Condition 1 (ii) implies $q_m^{AB} e_m \geqslant q_h^{AB} e_h$ so that $q \geqslant q_h^{AB}$. As a result, the offer loses money from AB borrowers.

   – An offer with $(\varphi e_h, q)$ is accepted by BB borrowers if and only if $qe_h \geqslant \bar{q}e_\ell$. By Condition 1 (iv), $\bar{q}e_\ell \geqslant q_h^{BB} e_h$ so that $q \geqslant q_h^{BB}$ and hence this offer loses money.

   – Since any offer $(\varphi e_h, q)$ that is accepted loses money on all types that accept it, B lenders cannot profit by offering large loans in stage 1.

This completes our characterization of this equilibrium.

F.4 Equilibrium Outcome 2: $\mathtt{lmh}$ with Cross-Subsidization

We construct an equilibrium with terminal loans φe_ℓ, φe_m, φe_h for BB-, AB-, and AA-borrowers, respectively, in which both AA- and AB-borrowers accept loans in the first stage. We then establish a set of sufficient conditions for it to be an equilibrium. We construct the equilibrium as follows.

F.4.1 On-Path Actions

Stage 1:

– G-class A-lenders offer φe_ℓ at $q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}$;

– F-class A-lenders offer φe_ℓ at q_m^{AB};

– B-lenders offer nothing;

– AA-borrowers accept a loan from a G-class lender;

– AB-borrowers accept a loan from an A-lender.

Stage 2:

– F-class A-lenders who observe stage-1 loan of $(\varphi e_\ell, q^A)$ from a G-class lender offer $\varphi(e_h - e_\ell)$ at q_h^{AA};

– Accepted-class A-lenders and B-lenders who see stage-1 loan of $(\varphi e_\ell, q^A)$ or $(\varphi e_\ell, q_m^{AB})$ offer $\varphi(e_m - e_\ell)$ at q_m^{AB};

– B-lender who sees no loan offers φe_ℓ at $q = \bar{q}$.

F.4.2 Equilibrium Payoffs

The payoffs to borrowers in equilibrium are as follows:

- AA-borrowers: $\varphi e_\ell q^A + \varphi\left(e_h - e_\ell\right) q_h^{AA}$;
- AB-borrowers with A signals from G-class lenders: $\varphi e_\ell q^A + (e_m - e_\ell) q_m^{AB}$;
- AB-borrowers with A signals from F-class lenders: $\varphi e_m q_m^{AB}$;
- BB-borrowers: $\varphi e_\ell \overline{q}$.

F.4.3 Equilibrium Conditions

Before we proceed with construction of beliefs and (off-path) strategies, we state necessary conditions on the model parameters so that our constructed equilibrium candidate is indeed an equilibrium. We later show that these conditions together with Assumption 1 are sufficient to ensure that relevant incentive constraints are satisfied.

Condition 2 *Suppose that the model parameters satisfy*

(i)

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} > 1 - \delta(1-\alpha),$$

or, equivalently, $q_h^{AA} > q_m^{AB}$. *Note that this condition is the reverse of Condition 1 (i) and is a sufficient condition for Condition 1 (v).*

(ii)

$$[1-\delta(1-\alpha)](e_m - e_\ell) \geqslant \delta\alpha(e_h - e_\ell),$$

or, equivalently, $q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell)$. *Note that this is the same condition as Condition 1 (ii).*

(iii)

$$\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2}(e_h - e_\ell) \geqslant \left[1 - \delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right](e_m - e_\ell),$$

or, equivalently, $q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell)$. *Note that this is the same condition as Condition 1 (iii).*

(iv)

$$e_\ell \geqslant \left(1 - \delta + \delta \frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right) e_m,$$

or, equivalently, $\bar{q} e_\ell \geqslant q_m^{BB} e_m$. *Note that by Assumption 1 (ii) and Corollary 1 we also have* $q_m^{BB} e_m \geqslant q_h^{BB} e_h$. *Combining, we have* $\bar{q} e_\ell \geqslant \max\left\{q_m^{BB} e_m, q_h^{BB} e_h\right\}$. *Note that this is the same condition as Condition 1 (iv).*

F.4.4 Beliefs

We classify out-of-equilibrium histories and beliefs based on the size of the stage-1 loan and the class of the lenders who made the loan.

1. *Small Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - Beliefs of G-class lenders when $\sigma^G = A$ are

$$\Pr(\sigma^F = A) = \begin{cases} \Pr(AA|A) & \text{if } q \geqslant q^A, \\ 0 & \text{if } q < q^A; \end{cases}$$

     - Beliefs of G-class lenders when $\sigma^G = B$ are

$$\Pr(\sigma^F = A) = \begin{cases} \Pr(AB|A) & \text{if } q > q_m^{AB}, \\ 0 & \text{if } q \leqslant q_m^{AB}; \end{cases}$$

     - Beliefs of F-class lenders are

$$\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant q^A, \\ 0 & \text{if } q < q^A. \end{cases}$$

   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.
     - Beliefs of G-class lenders are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^{AB}, \\ 0 & \text{if } q < q_m^{AB}; \end{cases}$$

– Beliefs of F-class lenders when $\sigma^F = A$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AA|A) & \text{if } q > q^A, \\ 0 & \text{if } q \leqslant q^A; \end{cases}$$

– Beliefs of F-class lenders when $\sigma^F = B$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AB|B) & \text{if } q \geqslant q^A, \\ 0 & \text{if } q < q^A. \end{cases}$$

2. *Medium Loans*

Define

$$\tilde{q}_{m1}: \quad \tilde{q}_{m1} e_m + q_h^{AA}(e_h - e_m) = q^A e_\ell + q_h^{AA}(e_h - e_\ell), \tag{A11}$$

$$\tilde{q}_{m2}: \quad \tilde{q}_{m2} e_m + q_h^{AA}(e_h - e_m) = q_m^{AB} e_m, \tag{A12}$$

$$\tilde{q}_{m3}: \quad \tilde{q}_{m3} e_m + q_h^{AB}(e_h - e_m) = q^A e_\ell + q_m^{AB}(e_m - e_\ell), \tag{A13}$$

$$\tilde{q}_{m4}: \quad \tilde{q}_{m4} e_m + q_h^{AB}(e_h - e_m) = q_m^{AB} e_m, \tag{A14}$$

$$\tilde{q}_{m5}: \quad \tilde{q}_{m5} e_m + q_h^{AA}(e_h - e_m) = q^A e_\ell + q_m^{AB}(e_m - e_\ell). \tag{A15}$$

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from a G-class lender.
  – Beliefs of G-class lenders when $\sigma^G = A$ are

$$\Pr(\sigma^F = A) = \begin{cases} \Pr(AA|A) & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

  – Beliefs of G-class lenders when $\sigma^G = B$ are

$$\Pr(\sigma^F = A) = \begin{cases} \Pr(AB|B) & \text{if } q > \tilde{q}_{m2}, \\ 0 & \text{if } q \leqslant \tilde{q}_{m2}; \end{cases}$$

  – Beliefs of F-class lenders when $\sigma^F = A$ are

$$\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

– Beliefs of F-class lenders when $\sigma^F = B$ are

$$\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m3}, \\ 0 & \text{if } q < \tilde{q}_{m3}. \end{cases}$$

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from an F-class lender.
  – Beliefs of G-class lenders when $\sigma^G = A$ are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

  – Beliefs of G-class lenders when $\sigma^G = B$ are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m4}, \\ 0 & \text{if } q < \tilde{q}_{m4}; \end{cases}$$

  – Beliefs of F-class lenders when $\sigma^F = A$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AA|A) & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

  – Beliefs of F-class lenders when $\sigma^F = B$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AB|B) & \text{if } q \geqslant \tilde{q}_{m5}, \\ 0 & \text{if } q < \tilde{q}_{m5}. \end{cases}$$

3. *Large Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_h, q)$. Then lenders' beliefs in this scenario going forward are irrelevant.

4. *No Loans*

   $\Pr(\sigma^- = A) = 0$.

F.4.5 Strategies

We now describe strategies beginning with borrowers' strategies in Stage 1 given any sets of offers arising from a single lender's deviation.

Borrowers' (off-path) Strategies in Stage 1

Define

$$\tilde{q}_{h1}: \quad \tilde{q}_{h1}e_h = q^A e_\ell + q_h^{AA}(e_h - e_\ell), \tag{A16}$$

$$\tilde{q}_{h2}: \quad \tilde{q}_{h2}e_h = q_m^{AB} e_m, \tag{A17}$$

$$\tilde{q}_{h3}: \quad \tilde{q}_{h3}e_h = q^A e_\ell + q_m^{AB}(e_m - e_\ell). \tag{A18}$$

1. *AA-Borrowers*

   Suppose a borrower observes at least $(N-1)$ offers $(\varphi e_\ell, q^A)$ from G-class lenders and at least $(N-1)$ offers $(\varphi e_\ell, q_m^{AB})$ from F-class lenders.

   - Suppose one lender offers $(\varphi e_\ell, q)$ where q is not prescribed by the equilibrium. The borrower's strategy is to accept the deviation offer if and only if $q > q^A$.
   - Suppose one lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q \geqslant \tilde{q}_{m1}$.
   - Suppose one lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q \geqslant \tilde{q}_{h1}$.

2. *AB-Borrowers with A signals from G-class lenders*

   Suppose a borrower observes $(N-1)$ offers $(\varphi e_\ell, q^A)$ from G-class lenders and no offers from F-class lenders.

   - Suppose one G-class lender offers $(\varphi e_\ell, q)$ where $q \neq q^A$. The borrower's strategy is to accept the deviation offer if and only if $q > q^A$.
   - Suppose one G-class lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{m3}$.
   - Suppose one G-class lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h3}$.

   Suppose a borrower observes N offers $(\varphi e_\ell, q^A)$ from G-class lenders and one offer from an F-class lender.

   - If the F-class lender's offer is $(\varphi e_\ell, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \max\{q_m^{AB}, \tilde{q}_{l1}\}$ where

$$\tilde{q}_{\ell 1}e_\ell + q_h^{AA}(e_h - e_\ell) = q^A e_\ell + q_m^{AB}(e_m - e_\ell).$$

- If the F-class lender's offer is $(\varphi e_m, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \max\{\tilde{q}_{m1}, \tilde{q}_{m5}\}$.[67]
- If the F-class lender's offer is $(\varphi e_h, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h3}$.

3. *AB-Borrowers with A signals from F-class lenders*

   Suppose a borrower observes one offer from a G-class lender and N offers $(\varphi e_\ell, q_m^{AB})$ from F-class lenders.

   - If the G-class lender offers $(\varphi e_\ell, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_m^{AB}$.
   - If the G-class lender offers $(\varphi e_m, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \min\{\tilde{q}_{m4}, \max\{\tilde{q}_{m2}, \tilde{q}_{m1}\}\}$.
   - If the G-class lender offers $(\varphi e_h, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h2}$.

   Suppose a borrower observes no offers from G-class lenders and $(N-1)$ offers $(\varphi e_\ell, q_m^{AB})$ from F-class lenders.

   - Suppose one F-class lender offers $(\varphi e_\ell, q)$ where $q \neq q_m^{AB}$. The borrower's strategy is to accept the deviation offer if and only if $q > q_m^{AB}$.
   - Suppose one F-class lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \min\{\tilde{q}_{m1}, \tilde{q}_{m4}\}$.[68]
   - Suppose one F-class lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h2}$.

4. *BB-Borrowers*

   Suppose a borrower observes at most one offer.

   - Suppose one lender offers $(\varphi e_\ell, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{l2}$, where

$$\tilde{q}_{\ell 2} e_\ell + q_m^{BB}(e_m - e_\ell) = \bar{q} e_\ell.$$

[67] Note, $\tilde{q}_{m5}$ is the price the borrower would accept if he can obtain an AA-priced loan from the G-class lenders. However, a price $\tilde{q}_{m1}$ is needed to ensure G-class lenders believe F-class lenders have an A signal.

[68] $\tilde{q}_{m1}$ is the price needed to "fool" F-class lenders about the G-class lenders' signal. Note that $\tilde{q}_{m1} \geqslant \tilde{q}_{m2}$ so that if the borrower has "fooled" F-class lenders such a loan is worth accepting. $\tilde{q}_{m4}$ is the price that justifies a BA borrower accepting the loan without "fooling" F-class lenders. In either case, the BA borrower would want to accept this loan.

- Suppose one lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \min\{\tilde{q}_{m6}, \max\{\tilde{q}_{m4}, \tilde{q}_{m7}\}\}$, where

$$\tilde{q}_{m6} e_m + q_h^{BB}(e_h - e_m) = \bar{q} e_\ell,$$
$$\tilde{q}_{m7} e_m + q_h^{AB}(e_h - e_m) = \bar{q} e_\ell.$$

- Suppose one lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > e_\ell / e_h$.

Lenders' Strategies in Stage 2 We next describe lenders' strategies for any credit history in stage 2 (i.e. any information set of lenders in stage 2).

1. *Small Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - When G-class lenders have signal $\sigma^G = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - When G-class lenders have signal $\sigma^G = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
     - Suppose F-class lenders have signal $\sigma^F = A$. If $q \geqslant q^A$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - Suppose F-class lenders have signal $\sigma^F = B$. If $q \geqslant q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$, and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.
     - Suppose G-class lenders have signal $\sigma^G = A$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - Suppose G-class lenders have signal $\sigma^G = B$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
     - When F-class lenders have signal $\sigma^F = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - When F-class lenders have signal $\sigma^F = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.

2. *Medium Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from a G-class lender.
     - When G-class lenders have signal $\sigma^G = A$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.
     - When G-class lenders have signal $\sigma^G = B$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.
     - Suppose F-class lenders have signal $\sigma^F = A$. If $q \geqslant \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AA})$ and if $q < \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.

  - Suppose F-class lenders have signal $\sigma^F = B$. If $q \geqslant \tilde{q}_{m3}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$, and if $q < \tilde{q}_{m3}$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.
- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from an F-class lender.
  - Suppose G-class lenders have signal $\sigma^G = A$. If $q \geqslant \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AA})$ and if $q < \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.
  - Suppose G-class lenders have signal $\sigma^G = B$. If $q \geqslant \tilde{q}_{m4}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$ and if $q < \tilde{q}_{m4}$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.
  - When F-class lenders have signal $\sigma^F = A$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.
  - When F-class lenders have signal $\sigma^F = B$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.

3. *Large Loans*

   If the borrower accepted a loan $(\varphi e_h, q)$ from any lender, all lenders offer $(0, 0)$.

4. *No Loans*

   If a lender has signal $\sigma = A$, they offer $(\varphi e_m, q_m^{AB})$. If a lender has signal $\sigma = B$, they offer $(\varphi e_\ell, \bar{q})$.

Borrowers' Strategies in Stage 2 For any history and any set of loan offers in Stage 2, the borrower accepts the loan with the highest qx.

Features of pricing thresholds Under our above assumptions, it is useful to note a few relationships between the various thresholds characterizing the off-equilibrium-path beliefs.

Lemma 2 *Suppose that Assumption 1 and Condition 2 are satisfied. Then the thresholds constructed in* (A11)-(A18) *satisfy the following conditions:*

(i) $\tilde{q}_{m1} > \tilde{q}_{m3} > \tilde{q}_{m4} > \tilde{q}_{m2}$,

(ii) $\tilde{q}_{m3} > \tilde{q}_{m5} > \tilde{q}_{m2}$,

(iii) $\tilde{q}_{m1} > \Pr(AA|A) q_h^{AA} + \Pr(AB|A) q_h^{AB}$,

(iv) $\tilde{q}_{h1} \geqslant \tilde{q}_{h3}$.

Proof:

(i) Inequality $\tilde{q}_{m4} > \tilde{q}_{m2}$ follows from $q_h^{AA} > q_h^{AB}$. Inequality $\tilde{q}_{m3} > \tilde{q}_{m4}$ follows from Condition 2 (i) and the definition of $q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}$. To show that $\tilde{q}_{m1} > \tilde{q}_{m3}$, rewrite (A11) and (A13) as

$$\tilde{q}_{m1}e_m = q^A e_\ell + q_h^{AA}(e_m - e_\ell),$$
$$\tilde{q}_{m3}e_m = q^A e_\ell + q_m^{AB}(e_m - e_\ell) - q_h^{AB}(e_h - e_m).$$

Using Condition 2 (i), we have $\tilde{q}_{m1} > \tilde{q}_{m3}$.

(ii) Inequality $\tilde{q}_{m3} > \tilde{q}_{m5}$ follows from Condition 2 (i). Inequality $\tilde{q}_{m5} > \tilde{q}_{m2}$ follows from Condition 2 (i) and the definition of q^A.

(iii) Using (A11),

$$\tilde{q}_{m1}e_m + q_h^{AA}(e_h - e_m) = q^A e_\ell + q_h^{AA}(e_h - e_\ell),$$
$$\tilde{q}_{m1}e_m = q^A e_\ell + q_h^{AA}(e_m - e_\ell),$$
$$\tilde{q}_{m1}e_m = \left[\Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}\right] e_\ell + q_h^{AA}(e_m - e_\ell).$$

Moreover, $\Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB} < q_h^{AA}$ by Condition 2 (i). Hence, $\tilde{q}_{m1} > \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_h^{AB}$.

(iv) Follows from Condition 2 (ii). □

F.4.6 Incentives

We now verify that given Assumption 1 and Condition 2, the strategies and beliefs described above constitute an equilibrium.

Borrowers' Stage-1 Deviations Consider first possible deviations by borrowers in stage 1. We will show that part (i) of Condition 2 together with Assumption 1 preclude them.

1. An AA-borrower could reject the stage-1 loan. Accepting is optimal as long as

$$q^A e_\ell + q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AB} e_m. \tag{A19}$$

Since $q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}$, $\Pr(AA|A) > \Pr(AB|A)$, and $q_h^{AA} > q_m^{AB}$

by Condition 2 (i), we have

$$q^A e_\ell + q_h^{AA}(e_h - e_\ell) > q_m^{AB} e_\ell + q_m^{AB}(e_h - e_\ell) = q_m^{AB} e_h > q_m^{AB} e_m.$$

Thus (A19) holds.

2. An AB-borrower with a stage-1 offer from F-class lenders could reject it and either obtain $(\varphi e_m, q_m^{AB})$ in stage 2 or obtain $(\varphi e_\ell, \bar{q})$ in stage 2. Accepting is optimal as long as

$$q_m^{AB} e_m \geqslant \bar{q} e_\ell. \tag{A20}$$

Note, since $q_m^{AB} = (1-\delta(1-\alpha))\bar{q}$, when $(1-\delta(1-\alpha))e_m > e_\ell$ as in Assumption 1 (i), this condition is satisfied.

3. An AB-borrower with a stage-1 offer from G-class lenders could reject it. Accepting is optimal as long as

$$q^A e_\ell + q_m^{AB}(e_m - e_\ell) \geqslant \max\{q_m^{AB} e_m, \bar{q} e_\ell\}. \tag{A21}$$

Under (A20), this incentive constraint reduces to $q^A \geqslant q_m^{AB}$ or $q_h^{AA} \geqslant q_m^{AB}$, which holds by Condition 2 (i).

Off-equilibrium path strategies specified above are constructed to be optimal for the borrower given prescribed continuation strategies.

Lenders' Stage-2 Deviations. We now analyze possible deviations of lenders in stage 2 and show that they are not profitable given Condition 2 (ii)-(iv).

1. *Small Loans*
   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - When G-class lenders have signal $\sigma^G = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
       * Any top-up to medium loan with $q \neq q_m^{AB}$ is either not accepted or unprofitable. Note, loans with $q > q_m^{AB}$ would be accepted with strictly positive probability but would earn negative expected profits. The reason is that for such prices with $q < q_h^{AA}$, only AB-borrowers accept making the loan unprofitable and at or above q_h^{AA}, both AA and AB's may accept making the loan unprofitable. Moreover, since borrowers in stage 2 accept the loan with the large qx, loan offers $(\varphi(e_m - e_\ell), q)$ with $q < q_m^{AB}$ are not accepted.

    * Any weakly profitable loan with a top up to e_h—and hence priced at most q_h^{AB}—is not accepted under Condition 2 (ii) which implies

$$q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell).$$

  - When G-class lenders have signal $\sigma^G = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$;
    * Any top-up to a medium loan with $q \neq q_m^{BB}$ is either not accepted or unprofitable.
    * Any weakly profitable loan with a top up to e_h is not accepted under Condition 2 (ii) which also implies

$$q_m^{BB}(e_m - e_\ell) \geqslant q_h^{BB}(e_h - e_\ell).$$

  - Suppose F-class lenders have signal $\sigma^F = A$. If $q \geqslant q^A$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
    * When $q \geqslant q^A$, any top-up to a large loan with $q \neq q_h^{AA}$ is either not accepted or unprofitable. Under Condition 2 (iii),

$$q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell),$$

      which implies any top-up to a weakly profitable medium loan is not accepted.
    * When $q < q^A$, any top-up to medium loan with $q \neq q_m^{AB}$ is either not accepted or unprofitable. Any weakly profitable loan with a top up to e_h again is not accepted under Condition 2 (ii).
  - Suppose F-class lenders have signal $\sigma^F = B$. If $q \geqslant q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$, and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
    * When $q \geqslant q^A$, any top-up to a medium loan $q \neq q_m^{AB}$ is either not accepted or unprofitable. Condition 2 (ii) implies that any weakly profitable top-up to a large loan is not accepted.
    * When $q < q^A$, any top-up to medium loan with $q \neq q_m^{BB}$ is either not accepted or unprofitable. Any weakly profitable loan with a top up to e_h again is not accepted under Condition 2 (ii).
- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.
  - Suppose G-class lenders have signal $\sigma^G = A$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.

- Suppose G-class lenders have signal $\sigma^G = B$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
- When F-class lenders have signal $\sigma^F = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
- When F-class lenders have signal $\sigma^F = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.

These strategies are optimal under Conditions 2 (ii)-(iii). The arguments are analogous to those for when the borrower accepted a small loan from a G-class lender.

2. *Medium Loans*

   Medium loans only occur off the equilibrium path. Note that all strategies in these histories involve topping up the borrower to a large loan that earns zero profits under the specified beliefs. Therefore, any deviation would either not be accepted or be unprofitable.

3. *Large Loans*

   Trivially, offering any further loans would result in non-repayment and negative profits.

4. *No Loans*

   - If a lender has signal $\sigma = A$, they offer $(\varphi e_m, q_m^{AB})$;
     - Any medium loan with $q \neq q_m^{AB}$ is unprofitable or not accepted.
     - Under Condition 2 (ii), any weakly profitable large loan is not accepted.
     - Under Assumption 1, any weakly profitable small loan is not accepted.
   - If a lender has signal $\sigma = B$, they offer $(\varphi e_\ell, 1)$.
     - Any small loan with $q \neq 1$ is unprofitable or not accepted.
     - Under Condition 2 (iv), $\bar{q}\varphi e_\ell \geqslant q_m^{BB}\varphi e_m$, so that weakly profitable medium loans are not accepted.
     - Condition 2 (ii) and (iv) together imply any weakly profitable large loan is not accepted.

Lenders' Stage-1 Deviations. We now present conditions such that for each signal, lenders' stage-1 strategies are optimal.

1. G-class lender with an A Signal.

- Any small loan at $q \neq q^A$ is either unprofitable or not accepted. At prices below q^A, borrowers do not accept. At prices above q^A, the offer would be accepted by both AAs and ABs. Such a loan necessarily loses money since it will be topped up in stage 2 in the same way as happens on the equilibrium path. Since the lender breaks even in equilibrium, offering higher prices must lose money.

- From Lemma 2 $\tilde{q}_{m1} > \tilde{q}_{m3}$. As a result, medium loans with $q \leqslant \tilde{q}_{m3}$ are not accepted. Medium loans with $q \in (\tilde{q}_{m3}, \tilde{q}_{m1})$ are accepted only by AB-borrowers. Condition 2 (ii) and $q^A > q_m^{AB}$ imply $q^A e_\ell + q_m^{AB}(e_m - e_\ell) > q_h^{AB} e_h$ or $\tilde{q}_{m3} > q_h^{AB}$. As a result, such a loan is accepted and yields negative expected profits. Medium loans with $q \geqslant \tilde{q}_{m1}$ are accepted by both AA and AB-borrowers. From Lemma 2, $\tilde{q}_{m1} > \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_h^{AB}$. As a result, any such loan must yield negative expected profits.

- From Lemma 2, large loans with $q < \tilde{q}_{h3}$ are not accepted. Large loans with $q \in [\tilde{q}_{h3}, \tilde{q}_{h1}]$ are accepted by only AB-borrowers. Since $\tilde{q}_{h3} > q_h^{AB}$ (same condition as for medium loans), such loans must be unprofitable. Large loans with $q \geqslant \tilde{q}_{h1}$ are accepted by both AA and AB-borrowers. Analogous to medium loans, $\tilde{q}_{h1} > \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_h^{AB}$ so that any such loan must yield negative expected profits.

2. F-class lender with an A Signal.

- Any small loan at $q \neq q_m^{AB}$ is either unprofitable or not accepted. At prices below q_m^{AB}, borrowers do not accept. At prices above q_m^{AB} but below q^A, only AB-borrowers accept and the loan is unprofitable. At prices above q^A, both AA- and BA-borrowers will accept and the loan is unprofitable.

- From Lemma 2 $\tilde{q}_{m1} > \tilde{q}_{m4}$. Medium loans with $q \leqslant \tilde{q}_{m4}$ are not accepted. Medium loans with $q \in (\tilde{q}_{m4}, \tilde{q}_{m1})$ are accepted only by AB-borrowers. Condition 2 (ii) implies $\tilde{q}_{m4} > q_h^{AB}$ so that such loans are unprofitable. Medium loans with $q \geqslant \tilde{q}_{m1}$ are unprofitable as described when a G-class lender with an A signal offers such a loan.

- From Lemma 2, large loans with $q < \tilde{q}_{h2}$ are not accepted. Large loans with $q \in (\tilde{q}_{h2}, \tilde{q}_{h1})$ are accepted by only AB-borrowers. Condition 2 (ii) implies $\tilde{q}_{h2} > q_h^{AB}$ so that such loans must be unprofitable. Large loans with $q \geqslant \tilde{q}_{h1}$ are accepted by both AA- and AB-borrowers. Analogous to large loans, $\tilde{q}_{h1} > \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_h^{AB}$ so that any such loan must yield negative expected profits.

3. Lender with a B Signal.

   - Any small loan with $q < q_{l2}$ is accepted by no borrowers. Any small loan with $q \in [q_{l2}, q_m^{AB})$ (if such an interval exists) is accepted only by a BB-borrower and must lose money (whether the BB type tops up to a large or a medium loan) because $q_{l2} > q_m^{BB}$, which follows from Condition 2 (iv). For any $q > q_m^{AB}$, the loan is accepted by BB-borrowers only or by AB- and BB-borrowers and these borrowers obtain a top up to a medium loan (or more) in stage 2. Such loans must earn negative profits.
   - Medium loans attract BB-borrowers if $q > \min\{\tilde{q}_{m4}, \tilde{q}_{m6}\}$. The lowest price loans that attract AB-borrowers satisfy $q > \tilde{q}_{m4}$. If the loan only attracts BB-borrowers, since $\min\{\tilde{q}_{m4}, \tilde{q}_{m6}\} > q_h^{BB}$, the loan must earn negative profits. If the loan attracts both BB and AB-borrowers, since $\tilde{q}_{m4} > q_h^{AB} > \Pr(AB|B)q_h^{AB} + \Pr(BB|B)q_h^{BB}$, the loan must earn negative profits.
   - Since $e_\ell > q_h^{BB} e_h$, large loans that attract only BB-borrowers must be unprofitable. Large loans must offer $q > \tilde{q}_{h2}$ to attract AB-borrowers. Since $\tilde{q}_{h2} > q_h^{AB}$, the loan earns negative profits (it attracts AB- and BB-borrowers).

This completes our characterization of this equilibrium.

F.5 Equilibrium Outcome 3: No Credit-History Building

We construct an equilibrium with no information aggregation. No offers are made (or accepted) in this equilibrium in stage 1. We then establish a set of sufficient conditions for it to be an equilibrium. We construct the equilibrium as follows.

F.5.1 On-Path Actions

- Stage 1:
  - Lenders make no offers (borrowers have no available action).
- Stage 2:
  - G-class A-lenders offer φe_m at $q_m^A = \Pr(AA|A)q_m^{AA} + \Pr(AB|A)q_m^{AB}$;
  - F-class A-lenders offer φe_m at q_m^{AB};
  - B-lenders offer φe_ℓ at $\bar{q}$;

– Borrowers accept the contract that yields the largest loan advance.

F.5.2 Equilibrium Payoffs

The payoffs to borrowers in equilibrium are as follows:

- AA-borrowers: $\varphi e_m q_m^A$;
- AB-borrowers with A signals from G-class lenders: $\varphi e_m q_m^A$;
- AB-borrowers with A signals from F-class lenders: $\varphi e_m q_m^{AB}$;
- BB-borrowers: $\varphi e_\ell \overline{q}$.

F.5.3 Equilibrium Conditions

Before we proceed with construction of beliefs and (off-path) strategies, we state necessary conditions on the model parameters so that our constructed equilibrium candidate is indeed an equilibrium. We later show that these conditions together with Assumption 1 are sufficient to ensure that relevant incentive constraints are satisfied.

Condition 3 *Suppose that the model parameters satisfy*

(i)

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2} \leqslant 1-\delta(1-\alpha),$$

or, equivalently, $q_h^{AA} \leqslant q_m^{AB}$. *Note that this is the same condition as part (i) of Condition 1.*

(ii)

$$[1-\delta(1-\alpha)](e_m - e_\ell) \geqslant \delta\alpha(e_h - e_\ell),$$

or, equivalently, $q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell)$. *Note that this is the same condition as part (ii) of Conditions 1 and 2.*

(iii)

$$\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}(e_h - e_\ell) \geqslant \left[1-\delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right](e_m - e_\ell),$$

or, equivalently, $q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell)$. *Note that this is the same condition as part (iii) of Conditions 1 and 2.*

(iv)

$$e_\ell \geqslant \left(1-\delta+\delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right)e_m,$$

or, equivalently, $\bar{q}e_\ell \geqslant q_m^{BB}e_m$. *Note that by Assumption 1 (ii) and Corollary 1 we also have* $q_m^{BB}e_m \geqslant q_h^{BB}e_h$. *Combining, we have* $\bar{q}e_\ell \geqslant \max\left\{q_m^{BB}e_m, q_h^{BB}e_h\right\}$. *Note that this is the same condition as part (iv) of Conditions 1 and 2.*

(v)

$$q_m^A(e_m-e_\ell) \geqslant q_h^A(e_h-e_\ell), \tag{A22}$$

where

$$q_m^A = \Pr(AA|A)\left[1-\delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right]+\Pr(AB|A)[1-\delta(1-\alpha)], \tag{A23}$$

$$\begin{aligned} q_h^A &= \Pr(AA|A)q_h^{AA}+\Pr(AB|A)q_h^{AB} \\ &= \Pr(AA|A)\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}+\Pr(AB|A)\delta\alpha, \end{aligned} \tag{A24}$$

and $\Pr(AA|A)$ *and* $\Pr(AB|A)$ *are given by* (A6)–(A7). *Notice that* (A22) *implies*

$$q_m^A e_m > q_h^A e_h. \tag{A25}$$

(vi)

$$\left[\Pr(AA|A)q_m^A+\Pr(AB|A)q_m^{AB}\right]e_m \geqslant q_h^A e_h,$$

where $q_m^{AB} = 1-\delta(1-\alpha)$, q_m^A *and* q_h^A *are given by* (A23)–(A24), *and* $\Pr(AA|A)$ *and* $\Pr(AB|A)$ *are given by* (A6)–(A7).

F.5.4 Beliefs

Beliefs of borrowers after offers are made in stage 1.

- If the borrower observes an offer from a G-class lender, then the borrower believes $\Pr(\sigma^G = A) = 1$.
- If the borrower observes an offer from an F-class lender, then the borrower believes $\Pr(\sigma^F = A) = 1$.

Beliefs of lenders at the end of stage 1. Note, for any accepted deviation loan in stage 1, beliefs of lenders in the same class as the lender who made the loan do not change. That is, for this class of lenders, beliefs are $\Pr(\sigma^- = A) = \Pr(\sigma^- = A|\sigma)$.

1. *Small loans.*
   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - Beliefs of F-class lenders are $\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^A, \\ 0 & \text{if } q < q_m^A. \end{cases}$
   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.
     - Beliefs of G-class lenders are $\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^{AB}, \\ 0 & \text{if } q < q_m^{AB}. \end{cases}$
   - Lenders do not update their beliefs about the other class' signal if the offer of a lender from their class was accepted.

2. *Medium loans.*
   - Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from a G-class lender.
     - Beliefs of F-class lenders are $\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^A, \\ 0 & \text{if } q < q_m^A. \end{cases}$
   - Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from an F-class lender.
     - Beliefs of G-class lenders are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}, \\ 0 & \text{if } q < \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}. \end{cases}$$

   - Lenders do not update their beliefs about the other class' signal if the offer of a lender from their class was accepted.

3. *Large loans.* Suppose the borrower has accepted a loan $(\varphi e_h, q)$. Then lenders' beliefs in this scenario going forward are irrelevant.

F.5.5 Strategies.

Stage-1 strategies of borrowers after offers are made in stage 1.

1. *Small loans.*

- Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from an G-class lender.
  - The borrower accepts if $q \geqslant q_m^A$, rejects otherwise.
- Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from an F-class lender. Define $\check{q}_\ell$ by

$$\begin{aligned}\check{q}_\ell e_\ell & + \Pr(AA|A) q_h^{AA}(e_h - e_\ell) + \Pr(AB|A) q_m^{AB}(e_m - e_\ell) \\ & = \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m.\end{aligned}$$

  - The borrower accepts if $q \geqslant \max\left\{\check{q}_\ell, q_m^{AB}\right\}$, rejects otherwise.

  Note that $\check{q}_\ell > q_m^{AB}$ and thus $\max\left\{\check{q}_\ell, q_m^{AB}\right\} = \check{q}_\ell$. Indeed, by Condition 3 (iii) and $q_m^{AA} > q_m^A$,

$$\check{q}_\ell e_\ell + \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right)(e_m - e_\ell) > \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m,$$

$$\check{q}_\ell > \Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB} > q_m^{AB}.$$

2. *Medium loans.*
   - Suppose the borrower observes an offer $(\varphi e_m, q)$ from a G-lender.
     - The borrower accepts the offer if and only if $q \geqslant \check{q}_m^G$, where

$$\check{q}_m^G e_m + q_h^A(e_h - e_m) \quad = \quad q_m^A e_m.$$

   - Suppose the borrower observes an offer $(\varphi e_m, q)$ from a F-lender.
     - The borrower accepts the offer if and only if $q \geqslant \check{q}_m^F$, where

$$\check{q}_m^F e_m + q_h^A(e_h - e_m) \quad = \quad \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m.$$

3. *Large loans.*
   - Suppose the borrower observes an offer $(\varphi e_h, q)$ from a G-class lender. The borrower accepts the loan if and only if $q e_h > q_m^A e_m$.
   - Suppose the borrower observes an offer $(\varphi e_h, q)$ from an F-class lender. The borrower accepts the loan if and only if $q e_h > \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m$.

Stage-2 strategies of lenders.

1. *Small loans.*

- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q \geqslant q_m^A$ from a G-class lender.
  - G-class A-lenders offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{BB})$ or nothing;
  - F-class A-lender offers $(\varphi(e_h - e_\ell), q_h^{AA})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{AB})$.
- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q < q_m^A$ from a G-class lender.
  - G-class A-lenders offer $(\varphi(e_m - e_\ell), q_m^{A})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{BB})$;
  - F-class A-lender offers $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{BB})$.
- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q \geqslant q_m^{AB}$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_\ell), q_h^{AA})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - F-class A-lender offers $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{BB})$ or nothing.
- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q < q_m^{AB}$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{BB})$;
  - F-class A-lender offers $(\varphi(e_m - e_\ell), q_m^{A})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{BB})$.

2. *Medium loans.*

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q \geqslant q_m^A$ from a G-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^{AB})$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{BB})$ or nothing;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^{AA})$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{AB})$.

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q < q_m^A$ from a G-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^A)$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{BB})$;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^{AB})$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{BB})$.
- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q \geqslant \left(\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right)$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^{AA})$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{AB})$;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^{AB})$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{BB})$ or nothing.
- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q < \left(\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right)$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^{AB})$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{BB})$;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^A)$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{BB})$.

3. *Large loans.* Suppose the borrower has accepted a loan $(\varphi e_h, q)$ in stage 1. Then lenders make no offers in stage 2.

F.5.6 Incentives.

Borrower's Stage-1 Deviations. Strategies of borrowers after offers are made in stage 1 are optimal given the specified continuation strategies of the lenders.

1. Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from a G-class lender.
   - It is optimal for the borrower to accept the offer if $q \geqslant q_m^A$. After doing so in stage 1, she believes she will receive (and accept) either an offer $(\varphi(e_h - e_\ell), q_h^{AA})$ from an F-class A-lender or an offer $(\varphi(e_m - e_\ell), q_m^{AB})$ from an F-class B-lender (or a G-lender). These follow from Condition 3 (ii) and (iii). Should she reject this stage 1 offer, she believes G-class lenders have an A signal and therefore will offer $(\varphi e_m, q_m^A)$. Condition 3 (iii) ensures that for all $q \geqslant q_m^A$, the borrower optimally accepts this stage 1 offer.

- It is optimal for the borrower to reject the offer if $q < q_m^A$. Should she accept this stage 1 offer, G-class lenders (who she believes have an A signal) will offer $(\varphi(e_m - e_\ell), q_m^A)$. The fact that these lenders will top her up to a medium rather than a large loan follows from Condition 3 (v). Instead, if she rejects the offer, the same G lenders will offer $(\varphi e_m, q_m^A)$ in the second stage. Since $q < q_m^A$, the borrower optimally rejects this stage 1 offer.

2. Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from an F-class Lender.

   - It is optimal for the borrower to accept the offer if $q \geqslant \check{q}_\ell$. After doing so in stage 1, she believes she will receive (and accept) either an offer $(\varphi(e_h - e_\ell), q_h^{AA})$ from a G-class A-lender or an offer $(\varphi(e_m - e_\ell), q_m^{AB})$ from a G-class B-lender (or an F-lender). These follow from Condition 3 (ii) and (iii). Should she reject this stage 1 offer, she believes F-class lenders have an A signal and therefore will offer $(\varphi e_m, q_m^{AB})$, and if G-class lenders also have an A signal, she will receive and accept an offer $(\varphi e_m, q_m^A)$. Thus the definition of $\check{q}_\ell$ ensures for any $q \geqslant \check{q}_\ell$, the borrower optimally accepts this stage 1 offer.
   - It is optimal for the borrower to reject the offer if $q < \check{q}_\ell$. For any $q \in [q_m^{AB}, \check{q}_\ell)$, the borrower's continuation payoffs from accepting or rejecting this stage 1 offer are the same as the case when $q \geqslant \check{q}_\ell$. Hence, by the definition of $\check{q}_\ell$, she optimally rejects such offers. Continuation payoffs from accepting this stage 1 offer when $q < q_m^{AB}$ are even lower, and it is thus optimal to reject such offers. Specifically, if a borrower accepts a stage 1 loan with $q < q_m^{AB}$ from an F-class lender, she believes she will receive an offer $(\varphi(e_m - e_\ell), q_m^{AB})$ in the second stage (with no chance of receiving an offer $(\varphi(e_h - e_\ell), q_h^{AA})$ which yields larger payoffs under Condition 3 (iii)).

3. Suppose the borrower observes an offer $(\varphi e_m, q)$ from a G-class lender.

   - It is optimal for the borrower to accept the offer if $q \geqslant \check{q}_m^G$. Note the definition of $\check{q}_m^G$ immediately implies $\check{q}_m^G \leqslant q_m^A$. After accepting the offer in stage 1, if $q \in [\check{q}_m^G, q_m^A)$, then the borrower believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^A)$ from a G-class lender. If instead $q \geqslant q_m^A$, she believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^{AA})$ from an F-class lender with an A signal or an offer $(\varphi(e_h - e_m), q_h^{AB})$ from an F-class lender with a B signal or a G-class lender. In either case, her expected payoff from accepting the stage 1 loan is $\varphi q e_m + \varphi q_h^A(e_h - e_m)$. For $q \geqslant \check{q}_m^G$, this payoff is larger than $\varphi q_m^A e_m$, which is her expected payoff from rejecting the stage 1 offer, and hence, it must be optimal for the borrower to accept the stage 1 offer.

- It is optimal for the borrower to reject the offer if $q < \check{q}_m^G$. Since $\check{q}_m^G < q_m^A$, should the borrower accept this stage 1 loan, she believe she will receive a payoff equal to $q\varphi e_m + q_h^A \varphi(e_h - e_m)$. When $q < \check{q}_m^G$ this payoff is smaller than the payoff she receives from rejecting the loan, $q_m^A \varphi e_m$ and hence rejecting the offer is optimal.

4. Suppose the borrower observes an offer $(\varphi e_m, q)$ from an F-class lender.

- It is optimal for the borrower to accept the offer if $q \geqslant \check{q}_m^F$. Note that the definition of $\check{q}_m^F$ immediately implies $\check{q}_m^F \leqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$. After accepting the offer in stage 1, if $q \in \left[\check{q}_m^G, \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right)$, then the borrower believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^A)$ from an F-class lender. If instead $q \geqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$, she believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^{AA})$ from a G-class lender with an A signal or an offer $(\varphi(e_h - e_m), q_h^{AB})$ from a G-class lender with a B signal or an F-class lender. In either case, her expected payoff from accepting the stage 1 loan is $\varphi q e_m + \varphi q_h^A(e_h - e_m)$. For $q \geqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$, this payoff is larger than $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] \varphi e_m$, which is her expected payoff from rejecting the stage 1 offer, and hence, it is optimal for the borrower to accept the stage-1 offer.
- It is optimal for the borrower to reject the offer if $q < \check{q}_m^F$. Since $\check{q}_m^F < \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$, should the borrower accept this stage 1 loan, she believes she will receive a payoff equal to $q\varphi e_m + q_h^A \varphi(e_h - e_m)$. When $q < \check{q}_m^F$ this payoff is smaller than the payoff she receives from rejecting the loan, $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right]$ and hence rejecting the offer is optimal.

5. Suppose the borrower observes an offer $(\varphi e_h, q)$ from a G-class lender.

- If the borrower accepts such an offer, she will receive no offers in the second stage. If she rejects, she believes she will receive an offer $(\varphi e_m, q_m^A)$ from G-lenders in the second stage. Hence, it is optimal to accept if and only if $q\varphi e_h > q_m^A \varphi e_m$.

6. Suppose the borrower observes an offer $(\varphi e_h, q)$ from an F-class lender.

- If the borrower accepts such an offer, she will receive no offers in the second stage. If she rejects, she with either receive an offer $(\varphi e_m, q_m^A)$ from G-lenders or an offer $(\varphi e_m, q_m^{AB})$ from F-lenders in the second stage. Hence, it is optimal to accept if and only if $q\varphi e_h > \left(\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right)\varphi e_m$.

Lenders' Stage-1 Deviations.

1. *Small loans.*
   - A G-class lender has no incentive to offer φe_ℓ.
     - Any offer below q_m^A is not accepted. Any offer above q_m^A is accepted. If F-lenders have an A signal, the borrower will be topped up to a large loan by Condition 3 (iii) and if they have a B signal the borrower will be topped up to a medium loan. This is implies that the probability of repayment of the loan is smaller than that priced into q_m^A. Hence, offers with prices $q \geqslant q_m^A$ earn negative expected profits.
   - An F-class lender has no incentive to offer φe_ℓ.
     - Any offer $q < \check{q}_\ell$ is not accepted. Any offer $q \geqslant \check{q}_\ell$ is accepted and earns negative expected profits. If the F-class lender has an A signal, then the expected payoff (per dollar of face value) is $-q + \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB} \leqslant -q + q_m^{AB} \leqslant 0$, where the first inequality follows from Condition 3 (i) and the second inequality follows from $\check{q}_\ell \geqslant q_m^{AB}$. If the F-class lender has a B signal, then the expected payoff (per dollar of face value) is

$$
\begin{aligned}
-q + \Pr(AB|B)q_h^{AB} + \Pr(BB|B)q_m^{BB} &\leqslant -q + \Pr(AB|B)q_m^{AB} + \Pr(BB|B)q_m^{AB} \\
&\leqslant -q + q_m^{AB} \leqslant 0.
\end{aligned}
$$

2. *Medium loans.*
   - A G-class lender has no incentive to offer φe_m.
     - Any offer $q < \check{q}_m^G$ is not accepted. Offers with $q \geqslant \check{q}_m^G$ yield negative expected profits. If the G-class lender has an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. Since $q \geqslant \check{q}_m^G$, the definition of $\check{q}_m^G$ implies $(q - q_h^A)e_m \geqslant q_m^A e_m - q_h^A e_h$. Condition 3 (v) implies $q_m^A e_m \geqslant q_h^A e_h$ and thus $-q + q_h^A \leqslant 0$. If the G-class lender has a B signal, their expected profits are weakly lower and hence also negative.
   - An F-class lender has no incentive to offer φe_m.

– Any offer $q < \check{q}_m^F$ is not accepted. Offers with $q \geqslant \check{q}_m^F$ yield negative expected profits. If the F-class lender has an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. Since $q \geqslant \check{q}_m^F$, the definition of $\check{q}_m^F$ implies $(q - q_h^A)e_m \geqslant \left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m - q_h^A e_h$. By Condition 3 (vi), $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m \geqslant q_h^A e_h$ and thus $-q + q_h^A \leqslant 0$. If the F-class lender has a B signal, their expected profits are weakly lower and hence also negative.

3. *Large loans.*

- A G-class lender has no incentive to offer φe_h.

  – Any offer with $qe_h \leqslant q_m^A e_m$ is not accepted. Offers with $qe_h > q_m^A e_m$ would be accepted by all borrowers and yield negative expected profits. If the G-class lender as an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. Since $qe_h > q_m^A e_m$ and Condition 3 (v) implies $q_m^A e_m \geqslant q_h^A e_h$, it follows that $qe_h > q_h^A e_h$. As a result, $-q + q_h^A < 0$. If the G-class lender has a B signal, their expected profits are weakly lower and hence also negative.

- An F-class lender has no incentive to offer φe_h.

  – Any offer with $qe_h \leqslant \left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m$ is not accepted. Offers with $qe_h > \left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m$ would be accepted by all borrowers and yield negative expected profits. If the F-class lender has an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. By Condition 3 (vi), $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m \geqslant q_h^A e_h$, which implies that $qe_h > q_h^A e_h$ so that the stage 1 offer earns negative expected profits. If the F-class lender has a B signal, their expected profits are weakly lower and hence also negative.

Borrower's Stage-2 Deviations. Trivially, accepting on-path stage-2 offers gives borrowers a strictly higher payoffs than rejecting them (recall that the lenders break even).

Lenders' Stage-2 Deviations. Stage-2 strategies of the lenders are optimal.

1. G-class lender with an A signal offers $(\varphi e_m, q_m^A)$.

- The lender cannot improve profits by offering a loan $(\varphi e_m, q)$ with $q \neq q_m^A$. If the lender offers a price $q > q_m^A$, all borrowers would accept the offer but it would yield negative expected profits (since q_m^A yields zero expected profits). If the lender offers a price $q < q_m^A$, the borrower would accept the loan.
- The lender cannot improve profits by offering a loan $(\varphi e_h, q)$. The highest price that a lender is willing to offer on a large loan is q_h^A. Condition 3 (v) implies that the borrower prefers $(\varphi e_m, q_m^A)$ to $(\varphi e_h, q_h^A)$, and so the lender does not have incentives to offer a large loan.
- The lender cannot improve profits by offering a loan $(\varphi e_\ell, q)$. Assumption 1 (i) implies that the borrower prefers $(\varphi e_m, q_m^A)$ to $(\varphi e_\ell, \bar{q})$, and since offering $q \geqslant \bar{q}$ can never be profitable, the lender does not have incentives to offer the small loan.

2. F-class lender with an A signal offers $(\varphi e_m, q_m^{AB})$.

   - The lender cannot improve profits by offering a loan $(\varphi e_m, q)$ with $q \neq q_m^{AB}$. If $q < q_m^{AB}$, the offer will not be accepted. If $q \in (q_m^{AB}, q_m^A)$, the offer would only be accepted by an AB borrower and hence earns negative expected profits (q_m^{AB} is the zero profit price for these borrowers). If $q \geqslant q_m^A$, all borrowers would accept the offer but it would yield negative expected profits (since q_m^A yields zero expected profits when all borrowers accept).
   - The lender cannot improve profits by offering a loan $(\varphi e_\ell, q)$ or $(\varphi e_h, q)$. The arguments are identical for those used for the G-class lender with an A signal.

3. Lenders with a B signal offer $(\varphi e_\ell, \bar{q})$. By Condition 3 (iv), the borrower prefers $(\varphi e_\ell, \bar{q})$ to $(\varphi e_m, q_m^{BB})$, and q_m^{BB} is the largest price that a lender with a B signal is willing to offer. So the lender has no incentives to deviate.

This completes our characterization of this equilibrium.